Physics 290 Lab 3
Cannons and 2D Motion

If they want peace, nations should avoid the pin-pricks that precede cannon shots.  
_Napoleon Bonaparte_

Your theory is crazy, but it is not crazy enough to be true.  
_Niels Bohr_

Let’s not go ballistic just yet….  
_Physics Lab Instructor_

**Introduction**

The study of ballistic motion, particularly for projectiles, has a long and storied history. Much of the work of Galileo and Newton was related to this topic as objects in free-fall were much less impacted by the dissipative forces that so confused Aristotle and other natural philosophers for nearly 2000 years. The “practical” application of this subject became apparent with the extensive use of gunpowder in warfare, and as a teenager Napoleon attended artillery school to learn what was then the cutting-edge science of ballistics. Some of the first “computers,” (actually very sophisticated mechanical gearboxes) were used in WWII to predict shell trajectories for battleships taking into account the curvature and rotation of the earth.

Because force is a vector, it can be decomposed into orthogonal components. Since gravity exerts a force towards the center of the earth, it is natural when considering the ballistic motion of a projectile to consider one axis aligned with the gravitational force and a second perpendicular to it. The problem of ballistic motion can then be “factorized” into two 1D problems, one with the force of gravity and one without.

In this lab, we will consider a problem that would have been familiar to Napoleon in his day. How can we predict the ballistic motion of a projectile from the fundamental knowledge of our launching device and the mass of our projectile? We will do this first using techniques which would have been familiar to Napoleon in his day: time-of-flight of the projectile and the apex of the trajectory. Since we now have better tools at our disposal, we will also perform a more complete analysis using a video camera and frame-grabbing software.

Ballistic motion is particularly simple if air resistance can be ignored. We will also try to see whether the effects of drag on our projectile can be observed.
Laboratory 3 Goals

- Understand the physical laws governing 2D ballistic motion.
- Categorize the properties of the launcher, and make predictions for ballistic motion based on these properties.
- Determine the launch angle needed to hit a target some arbitrary distance away.
- Gain familiarity with video analysis.
- Use video analysis to search for evidence of air resistance in our ballistic system.

IMPORTANT NOTE: Be sure to wear safety glasses around the launchers!

Description of Equipment

This lab is a bit less structured than the first lab in the sense that you have more options and possible avenues to achieve the lab goals. You should definitely spend a little time thinking through just exactly what you want to do before you do it. If you need some equipment which is not immediately available, do not hesitate to ask.

The central part of this lab is our “cannon,” actually a spring-based ball launcher. Please wear safety glasses when using this device and keep your head away from the business end of the launcher. We will have at least two masses of balls. For most of this lab you should use the steel balls.

The LabPro module (green box) allows a variety of sensors to be used together. In addition to the motion sensors, for this lab you might find a couple of microphones useful.

Finally, we will be using a video camera to record the motion of a projectile and analyzing this motion under Logger Pro. A separate handout describing the technical details of this process will be available.

Goal 1: Muzzle Velocity

The launcher is powered by a spring. When you compress the spring you are storing a certain amount of potential energy in the spring that is then released and transferred to the ball when the trigger is pulled. In order to determine what we expect the launcher to do under different conditions, we need to determine how much energy this launcher transfers to the projectile.

Energy is always conserved, and while some of the stored energy from the launcher will go into sound, most of it goes into making the ball move. The potential energy of the launcher is transferred into kinetic energy as the ball is launched. The kinetic energy of an object in motion is given by $ KE = \frac{1}{2} m v^2 $. If
we can measure the velocity of the ball at launch, we can determine the kinetic energy and hence the potential energy stored by the spring.

You may try the motion detectors if you wish, but they tend to not work well on an object as small as the projectile ball. Instead we will have to rely on some physics. There are a variety of measurements you can make which can be related to the initial velocity $v_0$. The first is the length of time the ball remains in the air. By measuring the “hang time” and considering the equations of motion for an object thrown up in the air, you should be able to determine $v_0$. You can try measuring this time with a stopwatch, although using microphones to measure the sound of the launcher firing and the ball hitting the table may be more accurate.

A second method for measuring $v_0$ is to consider the maximum height of the projectile after launch. At the apex of the projectile motion, the vertical component of velocity is zero. Where did the Kinetic Energy go? Moving against the earth’s gravitational field takes Work, (which is force integrated over some distance) and this work gives the ball a different kind of potential energy related to the gravitational field. This energy can be expressed as $\text{PE} = \text{mgh}$, where $g$ is the acceleration of gravity and $h$ is the change in height. When an object drops by a height $h$, it picks up kinetic energy equal to the change in potential energy. I will leave it up to you to devise a method for finding this height.

You should come up with a couple of specific methods for determining $v_0$. These measurements should be made with the launcher sending the ball as close to vertical as you can manage.

**Some Tasks:**

- Determine the initial velocity $v_0$ for a given launcher setting and ball at least two different ways. How reproducible are the results from launch to launch? Check how well these different determinations agree and calculate an uncertainty based on your data.

- How does the hang time or height depend upon the initial launcher angle? You should try to make the trajectory as vertical as possible, but this will never be perfect. Can you determine an uncertainty based on how close to vertical you are launching the ball?

- What would you expect for $v_0$ if you used a ball of a different mass? Make a prediction and *try it*.

**Goal 2: Predicting 2D Motion**

You will probably need to spend a little time calculating a few things before starting your experiments in this section. Based on the measurements made in the first part of this lab, you should now be able to predict where the projectile will
travel for any given launch angle. A more complicated problem is to predict what launch angle is needed to hit a given target some distance away, particularly if that target is at a different elevation (on the floor, for example).

To make your predictions, you will need to consider separately the horizontal and vertical position of the projectile as a function of time. You can neglect any kind of air resistance for this part (use the steel ball) and assume the only force acting on the ball in flight is gravity.

**Some Tasks:**

- For a given launch angle, predict the horizontal distance the ball will hit the table. Do not simply do this by trial and error, we want to use physics here. Use your previous measurement of $v_0$ and your understanding of 2D kinematics. Within uncertainties, do your results agree with your predictions? Try this for a couple of different angles.

- Now try to invert the problem. For a target a given horizontal distance away, can you determine the launch angle necessary to hit it? Is there only one solution? Note that you may not be able to write down a closed-form analytic solution to this problem, but that shouldn’t stop you from making predictions based on your physical theory. At the least, you can write down a table of ranges for given angles, then interpolate the angle required to hit a given distance. An Excel spreadsheet may prove useful here. Do this once for a target at the same elevation (on the table), and once for a target at a different elevation (on the floor). How well can you hit the target?

**Goal 3: A Closer Look**

The first two sections of this lab made certain assumptions about the nature of ballistic motion, including that gravity is the only force acting on the ball in flight. The experimental methods we used were also available to Napoleon in 1785. Here we will take a closer look at these assumptions using more modern equipment. Using the video camera, record two trajectories of the steel ball, one with a nearly vertical launch angle, and a second with a significant horizontal component. The exact launch angle will depend on how large of a field of view you can get through the camera, as you want to observe most (if not all) of the trajectory. If you have time, you should try a third launch using the lighter, wooden ball at the same large launch angle. A separate sheet will describe the technical details of recording and analyzing the video data. In the end, you should have columns of position and velocity for both x and y in LoggerPro from your video movies. Don’t forget to have a length reference in your movie, so you can calibrate the distances.
Some Tasks:

• Looking at the x and y velocity curves as a function of time, what can you say about the acceleration and forces in the x and y direction? Use the curve fit function to determine the acceleration from the velocity graph. Do these values agree with what you expect? What might be some sources of uncertainty here?

• From your data, calculate the initial velocity from the launcher and compare to your results in part 1. Is this a more or less accurate determination, and why?

• Think a little about the systematics of the video setup. How do you know how long one meter is? What would happen if the camera was rotated? How could you tell?

• Compare the data taken in the non-vertical launch with what you would predict from part 2. Does the trajectory and velocity of the ball match your prediction? Is this also true for the wooden ball?

• If you have time, analyze the wooden ball for evidence of air resistance. Drag acts as a velocity dependent force opposed to the direction of motion: $F_d = -kv$, where $k$ is some coefficient of drag. It is more difficult to see the effect of drag in the vertical direction, since the force of gravity is also acting there. Think about the component of drag in the horizontal direction, and see if there is any difference between the horizontal velocity for the wooden and metal ball. Why would mass make any difference here? If you really have time, you could even think about ways to make the force of drag larger, and try to demonstrate that with different launch conditions.

Conclusions

You have three weeks to complete this lab, and generally speaking each goal should take you a little under two hours to complete. Your lab book will contain a fair amount of analysis, particularly for goal 2, and you may not need much more than this to complete your notebook. Try to think a bit about uncertainties and examine the various issues which could be effecting the accuracy of your results. If you start to approach these labs thinking about sources of uncertainties, then you are starting to think like an experimentalist.

As stated before, the key to a successful lab is having a clearly described an rational approach to the goals described above. Getting the “right answer” is not the idea. Developing good experimental technique and exercising some creativity along with your understanding of physics to solve problems is the idea. I want to see that you understand the physics issues relevant to this lab, and that you have thought about your experiment and data in terms of these concepts.