A couple of pages from the note book of an outstanding Phys 290 student...

Goal Two: "Testing an Improved Model for the Motion of the Simple Pendulum"

Using the ~~same~~ same setup as in goal one, and taking 60 samples per second, we will vary the initial starting angle (θ) between very small, $\approx 3°$, and ~~very~~ fairly large, $\approx 65°$. We should be able to track exactly when the simple model of a pendulum breaks down as θ gets larger.

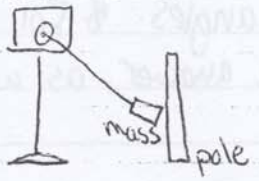
Trial	θ (rad)	$\sin\theta$	L (cm)	m (g)	Actual T (s)*	T simple model prediction (s)	T improved model prediction (s)**
1	0.05	0.05	25.5	100 g	1.010 ± 0.03	1.013	1.013
2	0.13	0.13	"	"	1.011 ± 0.04	"	1.014
3	0.26	0.257	"	"	1.014 ± 0.02	"	1.017
4	0.52	0.497	"	"	1.028 ± 0.02	"	1.030
5	0.72	0.66	"	"	1.041 ± 0.01	"	1.047
6	1.18	0.92	"	"	1.048 ± 0.03	"	1.070

* Period calculated from the frequency and error given by the FFT. See chart below:

Trial	ω (rad/s)	f (Hz)
1	6.220 ± 0.0003	0.98 ± 0.03
2	6.214 ± 0.0007	0.98 ± 0.04
3	6.195 ± 0.0004	0.97 ± 0.02
4	6.115 ± 0.0002	0.98 ± 0.02
5	6.030 ± 0.00053	0.96 ± 0.01
6	5.725 ± 0.0006	0.91 ± 0.03

** Calculated from the equation: $T = 2\pi\sqrt{l/g}\left(1 + \frac{1}{4}\sin^2(\theta/2) + \frac{9}{64}\sin^4(\theta/2)\right)$

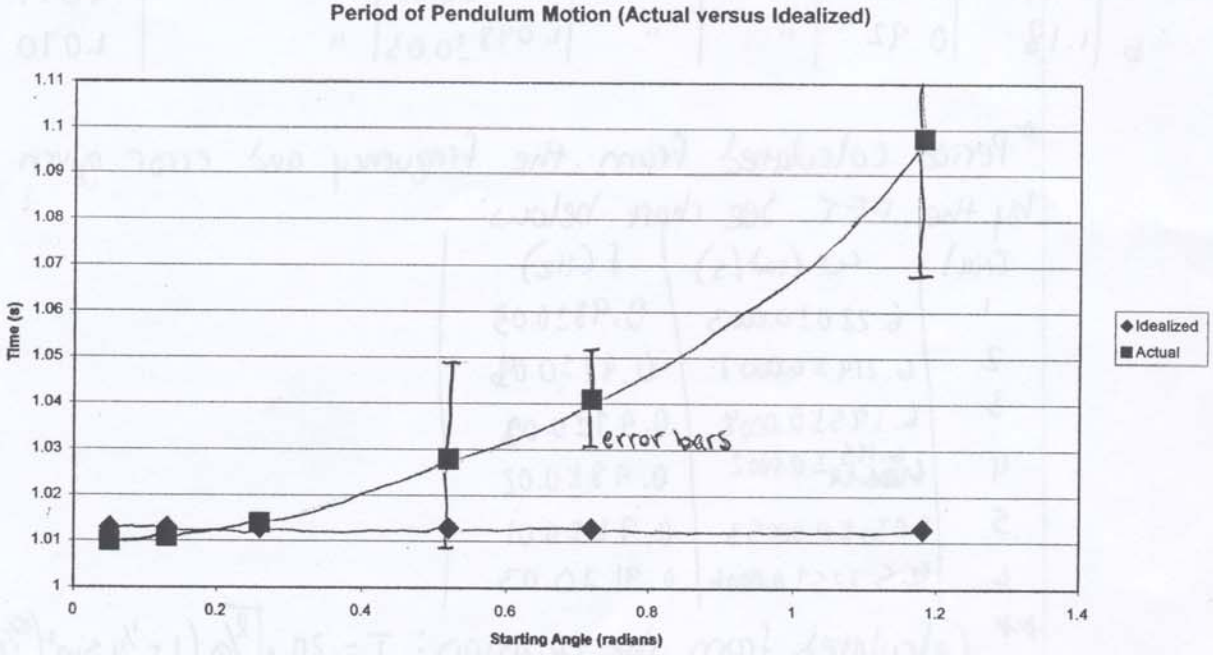
To test the repeatability of these results, we kept a set starting angle of 0.77 radians by placing the pendulum up against a pole to mark the starting position



The measured starting angle between the two ~~angles~~ trials did not deviate, and the measured ω and f were approximately the same as well, implying a good deal of accuracy in the system.

Trial	ω (rad/s)	f (Hz) from FFT	θ (rad)
1	6.000	$0.953\ Hz \pm 0.02$	0.77
2	5.998	$0.955\ Hz \pm 0.02$	0.77

If we graph the actual periods measured from pg 75 against the idealized, simple model of pendulum period prediction, we can see that the simple model no longer holds up for angles greater than 0.72 radians if not slightly less, aftering accounting for the estimated error of the actual period measurements.



Although not graphed here, it is possible to see that the improved model much more accurately predicts the actual period for large angles. & For small angles, the models closely match one another, as well as the actual data.