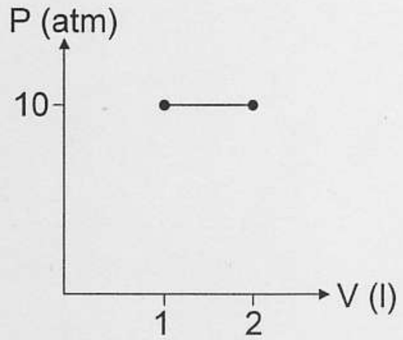
Name Key

PH353 – Foundations of Physics
Midterm Exam I, May 8, 2006

This is a closed book exam. However, you may use one page of notes. Please show your work. If you have questions about a problem, do not hesitate to ask! Additional paper is available, but if you use the back side of a page, please makes a note on the front.

1. (50) One liter of an ideal monatomic gas, initially at 300K, is heated and allowed to do expand quasistatically (doing work) as shown in the PV diagram at right.



(a) Find the work done.
(b) Find the final temperature.
(c) Find the heat added.
(d) Calculate the entropy change of the gas. Is this a reversible process?
(e) Calculate the overall efficiency of the process.

(a) $W = \text{area under graph} = -P(V_2 - V_1) = -(10\text{ atm} \times 1\times10^5 \frac{N}{m^2})(0.001\text{ m}^3)$

$[\Delta V = 1\ell = 0.001\text{ m}^3]$ $= -1000\text{ J}$ (NEGATIVE, BECAUSE WORK IS DONE ON WORLD)

(b) $PV = NkT \rightarrow V \propto T$ IF P CONSTANT

$\frac{T_f}{T_i} = \frac{V_f}{V_i} \Rightarrow T_f = 600\text{K}$

(c) $Q = \Delta U - W$ GET ΔU FROM $\frac{f}{2}\Delta(PV) = \frac{3}{2}P(V_2 - V_1)$

$= \frac{3}{2}(1000\text{ J}) = 1500\text{ J}$

$\therefore Q = 1500 - (-1000)\text{ J} = 2500\text{ J}$

(d) $\Delta S = \int \frac{dQ}{T} = \int_{T_i}^{T_f} \frac{C_p\,dT}{T} = C_p \ln\left(\frac{T_f}{T_i}\right)$

FOR MONATOMIC GAS $C_V = \frac{3}{2}Nk$, $C_p = \frac{5}{2}Nk$

$\Delta S = \frac{5}{2}Nk \ln 2$

[get Nk from initial conditions] $Nk = \frac{P_i V_i}{T_i} = \frac{1000\text{J}}{300\text{K}}$

$\therefore \Delta S = \frac{5}{2}\left(\frac{1000\text{J}}{300\text{K}}\right)\ln 2 = 8.33\text{ J/K}$

(e) $\epsilon = \frac{|W|}{Q_H} = \frac{1000\text{J}}{2500\text{J}} = 0.4$ (40%)

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2. (50) A cube of ice at 0° C, weighing 30 g, is added to a perfectly insulated cup containing 250 g of tea initially at 90° C.

(a) What is the final temperature?

As the system comes to equilibrium:

(b) Find the change in total entropy of the system.

(c) Estimate the change in the number of states available to the system (tea+ice).

(a) heat gained by ice $= 30\,g \times 80\,cal/g + (T_f - 0°C) \times 30\,g \times 1\,\frac{cal}{g°C}$

$= $ heat lost by tea $= 250\,g \times \frac{1\,cal}{g°C} \times (90°C - T_f)$

$$2400\,cal + 30\,\frac{cal}{°C} \cdot T_f = 250 \cdot 90\,cal - 250\,\frac{cal}{°C} T_f$$

$$280\,T_f = (22500 - 2400)\,cal \Rightarrow \boxed{T_f = 71.8\,°C}$$

$\sim 345\,K$

(b) $\Delta S_{system} = \Delta S_{ice\ melts} + \Delta S_{H_2O}(0° \to T_f) + \Delta S_{H_2O}(90° \to T_f)$

$$\Delta S_{ice} = \frac{mL_f}{T} = \frac{30\,g \cdot 80\,cal/g}{273\,K} \times 4.2\,J/cal = \underline{36.9\,J/K}$$

$$\Delta S_{H_2O} = \int_{T_i}^{T_f} \frac{mc\,dT}{T} = mc \ln(T_f/T_i)$$

$$\Delta S_{0° \to T_f} = 30\,g \cdot 1\,cal/g \cdot 4.2\,J/cal \ln\left(\frac{345}{273}\right) = \underline{29.4\,J/K}$$

$$\Delta S_{90° \to T_f} = 250\,g \cdot 1\,cal/g \cdot 4.2\,J/cal \ln\left(\frac{273+90}{345}\right) = \underline{-53.4\,J/K}$$

$\boxed{\Delta S_{SYSTEM}} = 36.9 + 29.4 - 53.4 = \underline{12.9\,J/K}$ (IRREVERSIBLE)

(c) FROM $S = k \ln \Omega$ $\quad \Delta S = k[\ln \Omega_f - \ln \Omega_i] = k \ln \frac{\Omega_f}{\Omega_i}$

So $\frac{\Omega_f}{\Omega_i} = e^{\Delta S/k}$

$$= e^{\frac{+12.9\,J/K}{1.38 \times 10^{-23}\,J/K}} = e^{9.3 \times 10^{23}}$$

WHICH IS A LARGE RATIO

THAT IS, REALLY LARGE.