PH353 Homework 2. Due Monday, April 17

I. The Poisson distribution is useful for events that have a very small probability of success, like the random decay of atomic nuclei or emission of a photon from a very dim light source. It can be derived by considering the toss of a very unfair coin: $p(heads) \ll 1$.

The exact formula for the probability of obtaining exactly n heads in N tosses of an unfair coin is:

$$P(N,n) = \frac{N!}{(N-n)!n!} p^n (1-p)^{N-n}$$

You should be able to derive this!

As shown in class, for the special case of p<<1, this can be simplified to

$$P(N,n) \approx \frac{\lambda^n}{n!} e^{-\lambda}$$

Where $\lambda = Np$ is the mean or expected number of events (in this case heads) in N trials. P(N,n) is called the Poisson distribution.

(a) Prove by direct summation that the mean number of events is:

$$< n > = \sum_{n=0}^{\infty} nP(N,n) = \lambda$$

(b) Prove by direct summation that the variance, or mean square deviation from the mean, is:

$$\sigma^{2} = <(n - < n >)^{2} > = -$$

Thus, the "standard deviation" is equal to the square root of the mean. This is the basis for the reported estimated error of, for example, public opinion polls.

II. Work problems 2.23, 2.24, 2.27 and 2.29 in Schroeder.