PH351 – Foundations of Physics
Midterm Exam II, November 18, 2005
Closed book/one page notes OK

If you have a question about the wording of any problem, please do not hesitate to ask the TA! If you need extra space, use the back of a page, or a blank sheet (available up front) but please make a note.

(1) [35 points] An object of mass \(m\) can move without friction along the \(x\) axis but is subjected to a spring force \(F = -sx\) in addition to a driving force \(F_0 \sin(\omega t)\). Its position therefore obeys the equation of motion

\[
\frac{d^2x}{dt^2} + sx = F_0 \sin(\omega t)
\]

(a) (15) Show how to solve this equation for \(x(t)\), assuming a steady state solution.
(b) (20) Sketch a graph of the amplitude of the motion \(A(\omega)\) as a function of the driving frequency \(\omega\). Defining \(\omega_0^2 = s/m\), discuss how the amplitude behaves for each of the three limiting or special cases: \(\omega \ll \omega_0\), \(\omega \gg \omega_0\) and \(\omega \sim \omega_0\).

(a) \[m \ddot{x} + sx = F_0 \sin \omega t\] \(\text{ASSUME } x(t) = A \sin \omega t \text{ (STEADY STATE)}\)

\[-m \omega^2 A \sin \omega t + sA \sin \omega t = F_0 \sin \omega t\]

\[
\Rightarrow A = \frac{F_0}{\sqrt{s - m\omega^2}} \text{ OR } \frac{F_0}{\sqrt{\omega_0^2 - \omega^2}} \text{ \text{WHERE } \omega_0^2 = \frac{s}{m}}
\]

(HAVE TO ADD TRANSIENT SOLUTION TO \(\ddot{x} + \omega_0^2 x = 0\) BUT NOT REQUIRED FOR EXAM)

(b) \[A(\omega)\]

\[
\begin{aligned}
\omega &< \omega_0 &\Rightarrow A &\approx \frac{F_0}{m \omega_0^2} \\
\omega &= \omega_0 &\Rightarrow A &\approx \frac{F_0}{\omega_0} \\
\omega &> \omega_0 &\Rightarrow A &\approx \frac{F_0}{m \omega^2} \rightarrow 0
\end{aligned}
\]

\(A\) is undefined for \(\omega = \omega_0\)

\(A\) is negative for \(\omega > \omega_0\)

\(\Rightarrow\) MOTION IS \(180^\circ\) OUT OF PHASE W.R.T. FORCE
(2) (30) Carbon monoxide, while not considered a greenhouse gas, is a polar molecule and absorbs light strongly in the infrared, at a wavelength of 4.7 µm. It is a linear molecule made of one carbon atom and one oxygen atom. Carbon has a mass of 12 amu while oxygen has a mass of 16 amu (1 amu = 1.6 x 10^{-27} kg).

(a) (10) Assuming that light travels as a wave with speed c = 3 x 10^8 m/s, from these data calculate the resonant frequency of linear oscillations of the C-O molecule.

(b) (20) Calculate the "stiffness" s of the bond connecting the C and the O.

\[
\text{(a) For light } \quad c = 3 \times 10^8 \text{ m/s and } \quad f = \frac{c}{\lambda} = \frac{3 \times 10^8 \text{ m/s}}{4.7 \times 10^{-6} \text{ m}} = \frac{6.38 \times 10^{13} \text{ Hz}}{4.7 \times 10^{-6} \text{ m}}
\]

\[
\text{(b) This is a "reduced mass" problem.}
\]

\[
S = \mu \omega^2 = \left( \frac{16 + 12}{16 + 12} \right) \left( 1.6 \times 10^{-27} \text{ kg} \right) \left( 2\pi \times 6.38 \times 10^{13} \text{ Hz} \right)^2
\]

\[
S = 1760 \text{ N/m}
\]

Note: To derive reduced mass

\[
m_1 \ddot{x} = -S(x - y)
\]

\[
m_2 \ddot{y} = -S(y - x)
\]

\[
\ddot{x} = -S \left( \frac{x - y}{m_1} \right)
\]

\[
\ddot{y} = -S \left( \frac{y - x}{m_2} \right)
\]

Assume \( \ddot{y} = \ddot{x} \)

\[
\omega^2 = \frac{S}{\mu} \quad \mu = \frac{m_1 m_2}{m_1 + m_2}
\]
(3) [35 points] In the picture below, an intrepid pulse moves towards a junction between dissimilar strings. To the left of the junction, the string has a linear density of \( \mu_L = 0.1 \) kg/m and to the right, a linear density of \( \mu_R = 0.2 \) kg/m. The string is stretched to a tension \( T = 40 \) N. Some time after the pulse hits the junction; there will be both a reflected and a transmitted pulse. In this problem we will ignore the reflected pulse.

![Graph showing a pulse over 1 mm and 1 cm.]

(a) (10) calculate the speed with which the pulse moves in the two strings.
(b) (10) calculate the width of the transmitted pulse in cm. (Hint: consider that amount of time that it takes the incoming pulse to cross the junction. The width on the right is not 1 cm!)
(c) (15) calculate the height of the transmitted pulse.

\[
(a) \quad C_L^2 = \frac{T}{\mu_L} \quad C_L = \sqrt{\frac{T}{\mu_L}} = \sqrt{\frac{40 \text{ N}}{0.1 \text{ kg/m}}} = 20 \text{ m/s}
\]
\[
C_R = \sqrt{\frac{T}{\mu_R}} = \sqrt{\frac{40 \text{ N}}{0.2 \text{ kg/m}}} = 14.1 \text{ m/s}
\]

(b) AT THE JUNCTION a PULSE TAKES \( \Delta t = \frac{0.01 \text{ m}}{0.01 \text{ m/s}} = 1 \text{ s} \) TO CROSS

OUTGOING PULSE APPEARS IN \( 5 \times 10^{-4} \) s

\( W = 5 \times 10^{-4} \times 14.1 \text{ m/s} \)

\( W = 0.705 \text{ cm} \)

(c) TRANSMISSION COEFFICIENT \( = \frac{Z_2}{Z_1 + Z_2} = \frac{2}{\frac{T_1}{C_1} + \frac{T_2}{C_2}} \)

\( C_T = \frac{2}{1 + \frac{T_1}{C_1} / \frac{T_2}{C_2}} = 0.827 \text{ (mm)} \) SINCE INCOMING HEIGHT = 1 mm