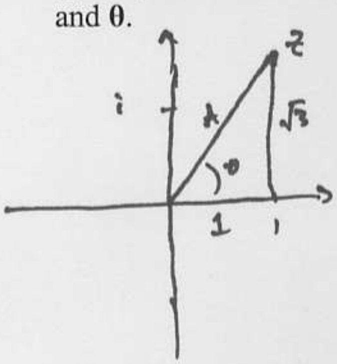
Name ~~~~Key~~~~

PH351 – Foundations of Physics
Midterm Exam I, October 21, 2005

Closed book/1 page notes OK. Additional blank sheets of paper are available up front. **Please show your work and please put your name on each page, in case they become separated. Thanks and good luck!**

1. Let the complex number **z = 1 + i√3. Hint:** for the following, it may be helpful to make a diagram of the complex plane, showing **z** and its components.

(a) (10) Show how to write **z** in the form $\mathbf{Ae^{i\theta}}$ and determine numerical values for **A** and **θ**.



$A = \sqrt{x^2+y^2} = \sqrt{1^2 + (\sqrt{3})^2} = 2$

$\tan\theta = y/x = \sqrt{3} \qquad \theta = \pi/3 \quad (60°)$

$\underline{z = 2e^{i\pi/3}}$

(b) (15) Find $\mathbf{z^2}$ and **1/z** and write these numbers in the form $\mathbf{Ae^{i\theta}}$.

$z^2 = \left(2e^{i\pi/3}\right)^2 = 4e^{i2\pi/3}$

$\frac{1}{z} = \frac{1}{2e^{i\pi/3}} = \frac{1}{2}e^{-i\pi/3}$

(c) (10) Express the reciprocal of **1+i**, or **1/(1 + i)**, in the form **x + iy**.

$\frac{1}{1+i} \cdot \frac{1-i}{1-i} = \frac{1-i}{2} = \underline{\frac{1}{2} - \frac{1}{2}i}$

ALTERNATIVELY SINCE $1+i = \sqrt{2}e^{i\pi/4}$

$\frac{1}{1+i} = \frac{1}{\sqrt{2}}e^{-i\pi/4} = \frac{1}{\sqrt{2}}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin(-\pi/4)\right)$

$= \frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}\right) = \underline{\underline{\frac{1}{2}(1-i)}}$

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(2) [30 points] A torsional oscillator has a mass with moment of inertia **I** which can rotate without friction about a point, however it is subject to a restoring torque $\boldsymbol{\tau} = -s\boldsymbol{\theta}$. Its angular position θ thus obeys the equation of motion

$$I\ddot{\theta} + s\theta = 0 .$$

The mass is initially at rest at $\theta = \theta_0$, and is then released. Find the subsequent rotational oscillation $\theta(t)$ of the mass in terms of I, s and θ_0.

rewrite equation as $\ddot{\theta} + \frac{s}{I}\theta = 0$

has solution $\theta = A\cos\left(\sqrt{\frac{s}{I}}\,t + \phi\right)$

subject to $\theta(0) = \theta_0 = A\cos\phi$

$\dot{\theta}(t) = -\sqrt{\frac{s}{I}}\sin\left(\sqrt{\frac{s}{I}}\,t + \phi\right)$

$\dot{\theta}(0) = 0 = -\sqrt{\frac{s}{I}}\sin\phi$

So we conclude $\sin\phi = 0 \Rightarrow \phi = 0$

$\theta = \theta_0\cos\left(\sqrt{\frac{s}{I}}\,t\right)$ IS THE COMPLETE SOLUTION

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(3) A swinging "pet door" has mass m = 0.2 kg and is attached to a spring of stiffness s = 1.8 kg/s^2. A dampener supplies a frictional force $-b$ x *velocity* with b to be determined later. The door position is given by **x**, which obeys the equation of motion:

$$m\frac{d^2x}{dt^2}+b\frac{dx}{dt}+sx=0.$$

(a) [25 points] Show how to solve the equation of motion and give its general solution, which contains two adjustable parameters in addition to the non-adjustable parameters determined by m, b and s.

rewrite as $\ddot{x}+\frac{b}{m}\dot{x}+\frac{s}{m}x=0$ AND $\gamma\equiv\frac{b}{m}$, $\omega_0^2=\frac{s}{m}$

$\rightarrow \ddot{x}+\gamma\dot{x}+\omega_0^2x=0$

SUBSTITUTE $x(t)=Ce^{pt}$ WHERE C, p may be complex #'s

$\dot{x}=pCe^{pt}$ $\ddot{x}=p^2Ce^{pt}$

$\rightarrow p^2Ce^{pt}+\gamma pCe^{pt}+\omega_0^2Ce^{pt}=0.$ since $Ce^{pt}\neq 0$

$\rightarrow p^2+\gamma p+\omega_0^2=0$ OR $p=-\frac{\gamma}{2}\pm\sqrt{(\frac{\gamma}{2})^2-\omega_0^2}$

choose $p_1=-\frac{\gamma}{2}+\sqrt{(\frac{\gamma}{2})^2-\omega_0^2}$

$p_2=-\frac{\gamma}{2}-\sqrt{(\frac{\gamma}{2})^2-\omega_0^2}$

GENERAL SOLUTION IS

$x(t)=C_1e^{p_1t}+C_2e^{p_2t}$ C_1, C_2 to be determined FROM INITIAL CONDITIONS

(b) [15 points] A cat, being chased by a large dog, hits the pet door at $t = 0$ with $x = 0$ and dx/dt = 3 m/s. Find **b** such that the door closes as quickly as possible, without oscillation (the so-called "critical damping case").

"CRITICAL DAMPING" HAPPENS WHEN $\sqrt{(\frac{\gamma}{2})^2-\omega_0^2}=0$

$(\frac{\gamma}{2})^2=\omega_0^2=\frac{s}{m}$ (INDEPENDENT OF INITIAL CONDITIONS)

$(\frac{b}{2m})^2=\frac{s}{m}\rightarrow b=2\sqrt{sm}$

Substitute $s=1.8$ Kg/s^2, $m=0.2$ Kg

$b=2\sqrt{1.8\frac{Kg}{s^2}\cdot 0.2Kg}=\boxed{1.2\ Kg/s}$