PH 351 11/23/05 LECTURE NOTES

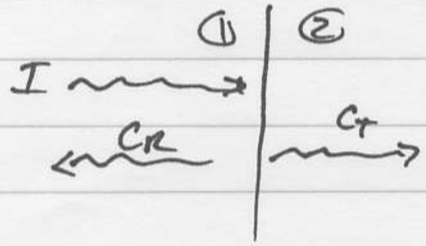
REVIEW OF PREVIOUS LECTURE (STRINGS, AGAIN).

- DERIVED $\frac{\text{ENERGY TRANSPORTED}}{\text{UNIT TIME}}$ = POWER TRANSMITTED

$$P = \frac{1}{2} Z \omega^2 A^2 \qquad \left\{ \begin{array}{l} A = \text{AMPLITUDE} \\ Z = \text{IMPEDANCE} \end{array} \right.$$

$$= (\text{WATTS})$$

- CONSIDERING ENERGY REFLECTED / TRANSMITTED AT BOUNDARY



$$C_R = \frac{Z_1 - Z_2}{Z_1 + Z_2}$$

$$C_T = \frac{2Z_1}{Z_1 + Z_2}$$

$$\frac{\text{ENERGY REFLECTED}}{\text{INCIDENT}} = \left(\frac{Z_1 - Z_2}{Z_1 + Z_2}\right)^2 \qquad \frac{\text{ENERGY TRANSMITTED}}{\text{INCIDENT}} = \frac{4Z_1Z_2}{(Z_1 + Z_2)^2}$$

→ TODAY

SHOW IN LECTURE THAT:

$$P = \frac{1}{2} Z \omega^2 A^2 = \frac{\text{ENERGY}}{\text{UNIT LENGTH}} \times \text{VELOCITY OF WAVE}$$

$$= \frac{1}{2} \mu \omega^2 A^2 \times c$$

$$= \frac{1}{2} Z \omega^2 A^2 \quad \text{because } Z = \mu c$$

The $\frac{1}{2} \mu \omega^2 A^2$ is THE TOTAL ENERGY/UNIT LENGTH OF STRING SEGMENT. (SHM)

(2)

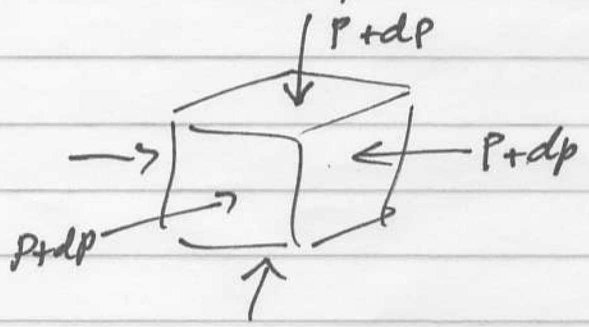
DERIVATION OF WAVE EQUATION FOR FLUIDS

REQUIRE <u>ONLY</u> THAT "RESTORING FORCE" $\propto$ (−) "DISPLACEMENT"

$\Rightarrow$ GET WAVE EQUATION

<u>BULK MODULUS</u> = RESPONSE OF FLUID TOWARD COMPRESSION BY EXCESS PRESSURE

CONSIDER FLUID, VOLUME V_0, DENSITY ρ (IN A BOX)



define $B = \dfrac{-dP}{dV/V_0}$

dV = VOLUME CHANGE, NORMALIZED TO V_0

B IS POSITIVE CONSTANT. MINUS SIGN CORRECTS FOR NEGATIVE VOLUME CHANGE UPON PRESSURE INCREASE

$$B = -V_0\, dP/dV$$

(CONTINUE NEXT PAGE).

B has dimensions of pressure N/m^2

$B > 0$ BECAUSE VOLUME DECREASES WHEN PRESSURE ↑

$\boxed{B\ (\text{AIR}) \sim 1.41 \times 10^5\ N/m^2}$ ADIABATIC

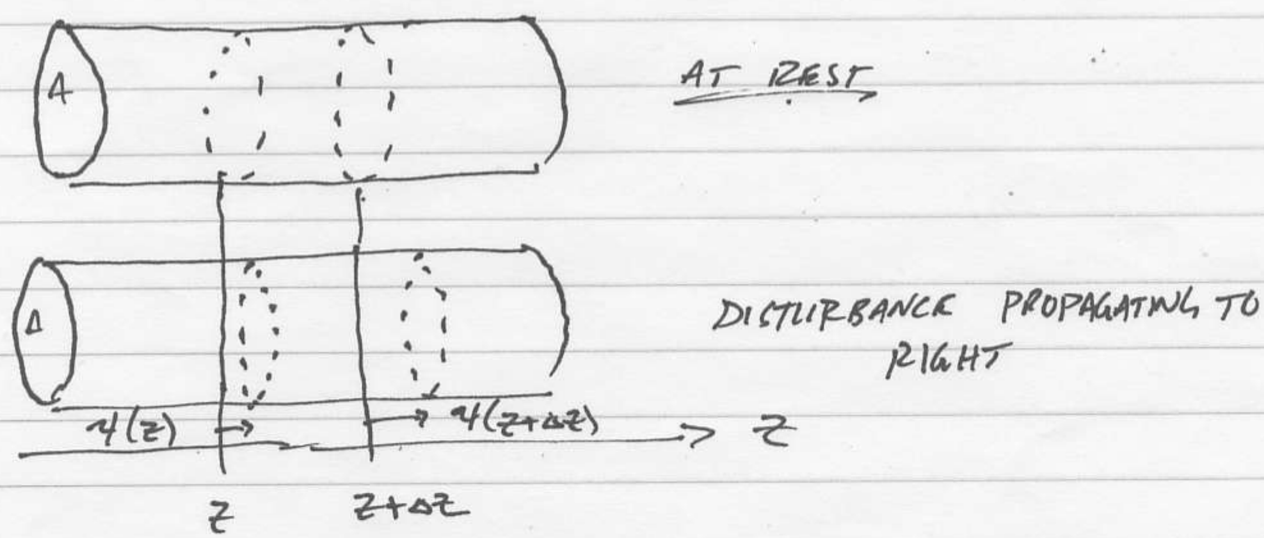
$\boxed{\text{cf ATM. PRESSURE} = 1.01 \times 10^5 \frac{N}{m^2}}$

ALSO MEASURE COMPRESSIBILITY $K = \frac{1}{\rho}\frac{d\rho}{dP} = \frac{1}{B}$

$\Rightarrow$ (VOLUME REACTS TO PRESSURE INCREASE BY EXERTING PRESSURE $P = -\frac{B}{V_0}\, dP$)

NOTE TWO VALUES FOR B $\left\{ \begin{matrix} \text{ISOTHERMAL} \\ \text{ADIABATIC} \end{matrix} \right\}$. (WILL DERIVE · PH 353)

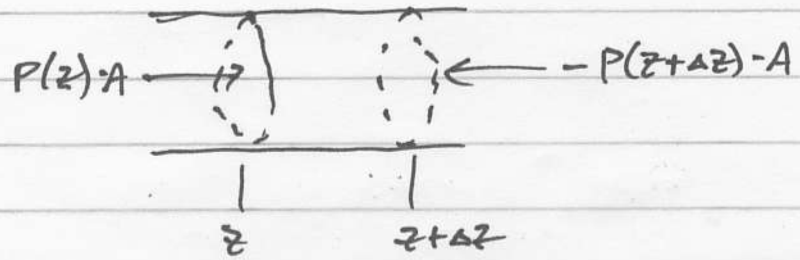
WAVE EQUATION: FLUID PLUG IN TUBE, AREA A



$\psi(z)$ MEASURES DISPLACEMENT OF PLUG FROM EQUILIBRIUM POSITION

PLUG HAS VOLUME $A\Delta z$, DENSITY ρ (kg/m^3)

NET FORCE ON PLUG



$$\text{NET FORCE} = -A\left(P(z+\Delta z) - P(z)\right) \cdot \frac{\Delta z}{\Delta z}$$

$$= -A\left(\frac{P(z+\Delta z) - P(z)}{\Delta z}\right)\Delta z$$

$$= -V\frac{\partial P}{\partial z}$$

NEWTON'S 2^{ND} LAW

$$M\frac{\partial^2 \psi}{\partial t^2} = -V\frac{\partial P}{\partial z} \qquad \text{but } \rho = \frac{M}{V}$$

$$\frac{\partial^2 \psi}{\partial t^2} = -\frac{1}{\rho}\frac{\partial P}{\partial z}$$

HOW DOES $P(z)$ DEPEND ON $\psi(z)$?

NOTE: IF $\psi(z) = \psi(z+\Delta z)$, PLUG MOVES BUT VOLUME DOES NOT CHANGE

IF $\psi(z) \neq \psi(z+\Delta z)$, $\Big\{$ VOLUME, DENSITY AND PRESSURE ALL CHANGE!

ASSUME PRESSURE CHANGE IS SMALL AND EXPAND AS TAYLOR SERIES:

"AMBIENT" "EXCESS PRESSURE"

$$P(V) \sim P_0 + \frac{dP}{dV}\cdot\Delta V = P_0 + \Delta P$$

FROM DEFINITION OF BULK MODULUS

$$B = -V_0\frac{dP}{dV} \Rightarrow \frac{dP}{dV} = -\frac{B}{V_0}$$

$$P(V) \sim P_0 + \frac{B}{V_0}\Delta V$$

ΔV depends on $\psi(z)$

$$\Delta V = \psi(z+\Delta z) - \psi(z)\cdot A$$

$$\Delta V = \frac{A\,\Delta z}{\cancel{\Delta z}}\left(\frac{\psi(z+\Delta z)-\psi(z)}{\Delta z}\right) = \frac{\cancel{A}}{\cancel{\Delta z}}\frac{\partial\psi}{\partial z}\cdot V_0$$

$$\therefore\ P(V) = P_0 - \frac{B}{V_0}\cdot\frac{\partial\psi}{\partial z}V_0 = P_0 - B\frac{\partial\psi}{\partial z}$$

$$\frac{\partial^2\psi}{\partial t^2} = -\frac{1}{\rho}\frac{\partial}{\partial z}\left(P_0 - B\frac{\partial\psi}{\partial z}\right)$$

$$\boxed{\frac{\partial^2\psi}{\partial t^2} = \frac{B}{\rho}\frac{\partial^2\psi}{\partial t^2}}$$

WAVE EQUATION

~~WE SEE~~

WE SEE IMMEDIATELY THAT

$\frac{B}{\rho}$ has dimensions of (velocity)^2

$$c = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{1}{K\rho}}$$

cf STRING $c = \sqrt{\frac{T}{\mu}}$ $\quad B \leftrightarrow T$, $\rho \leftrightarrow \mu$

HOW ABOUT IMPEDANCE? $Z = B/c = \frac{\text{FORCE}}{\text{AREA}} / \text{SPEED}$

DOES NOT CORRESPOND TO $Z = T/c$ (STRING)

$\Rightarrow$ THIS IS BECAUSE WE HAVE CONSTRAINED FLUID TO TUBE OF AREA A. $Z = B/c$ = IMPEDANCE/UNIT AREA $= \rho c$

TOTAL IMPEDANCE $= Z \cdot A$ (SEE WHY THIS IS IMPORTANT LATER ON)

$\Rightarrow$ SINCE AREA IS UNDEFINED FOR LONGITUDINAL WAVE PROPAGATING IN FLUID, MOST QUANTITIES ARE IN TERMS OF "PER AREA"

THEN: $C_R = \frac{Z_1 - Z_2}{Z_1 + Z_2}$ $\quad C_T = \frac{2Z_1}{Z_1 + Z_2}$

ENERGY TRANSPORT $= \frac{1}{2} Z \omega^2 A^2$ $\Big\{$ WATTS PER UNIT AREA

= INTENSITY