PH351- Homework 9. Due Monday, Nov. 28

1. ***An important problem in many branches of engineering and physics.*** A stretched string has mass per unit length $\boldsymbol{\mu}$ and tension $\mathbf{T}$. At its right end, at $\mathbf{x = L}$, the string is tied to a massless ring that is free to slide up and down on a frictionless rod. Thus the ring provides zero vertical force, but it provides a force to the right equal to the string tension $\mathbf{T}$. Now we add a "damper" attached to the right end of the string that provides a vertical force on the right end of the string equal to

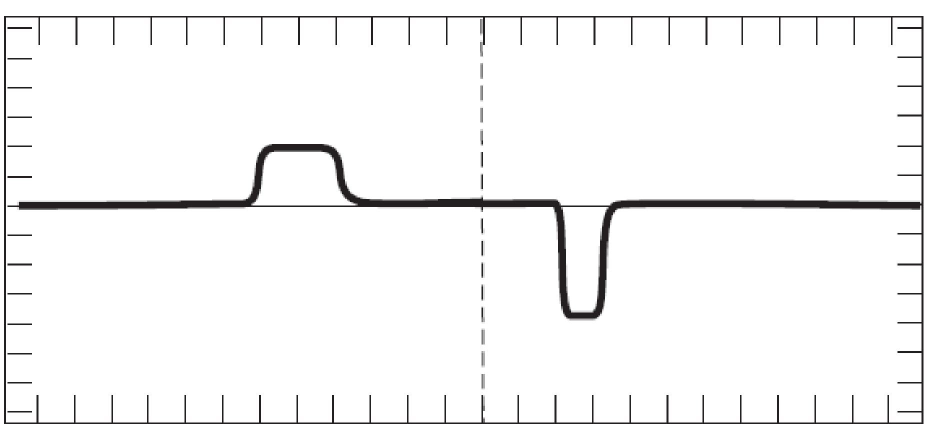
$$F_y = -\gamma\, \partial\psi(t, L)/\partial t$$

Where $\boldsymbol{\psi(t,x)}$ is the transverse displacement of a point at $\mathbf{x}$ on the string. If a right moving wave comes along the string, when the wave comes to the end it will in general create a left-moving reflecting wave. Thus the total wave is

$$\psi(t, x) = f(t - x/c) + C_R\, f(t + x/c - 2L/c).$$

Find the reflection coefficient $\mathbf{C_R}$ in terms of $\boldsymbol{\mu}$, $\mathbf{T}$, and $\boldsymbol{\gamma}$. What value of $\boldsymbol{\gamma}$ will cause $\mathbf{C_R}$ to vanish? In other words, $\mathbf{C_R}$ can be chosen such that there is no reflection. The term "$\mathbf{-2L/c}$" term in the reflected wave is there so that the two functions match at $\mathbf{x = L}$: $\mathbf{f(t-x/c)}$ becomes $\mathbf{f(t-L/c)}$ while $\mathbf{f(t + x/c - 2L/c)}$ becomes $\mathbf{f(t + L/c - 2L/c) = f(t - L/c)}$. Previously, we took $\mathbf{L = 0}$ so this term did not appear.

2. ***The Mystery Pulse.*** A long string stretched along the $\mathbf{x}$ axis has a constant tension but a mass per unit length that is different on the left hand part than in its right hand part. Part of the string is shown below. The dashed line is the boundary where the density changes. In the past, the displacement of the string was zero except for a more-or-less rectangular pulse traveling from left to right. When this pulse arrived at the boundary, a reflected pulse moving to the left and a transmitted pulse traveling to the right emerged. A short time afterward, a photograph of the string was taken. The figure below shows the photograph. The pulse on the left, of height 2 units and length 2 units is the *reflected* pulse. The pulse on the right, of height −4 units and length 1 unit, is the *transmitted* pulse. Describe the original pulse, giving its height and width and saying whether its height was positive or negative.



3. The compressibility of water is $\boldsymbol{\kappa}_\mathbf{w} = 4.9 \times 10^{-10}\ \mathrm{m^2\,N^{-1}}$. The compressibility of air is $\boldsymbol{\kappa}_\mathbf{a} = 7.1 \times 10^{-6}\ \mathrm{m^2\,N^{-1}}$. The density of water is $\boldsymbol{\rho}_\mathbf{w} = 1.0 \times 10^3\ \mathrm{kgm^{-3}}$. The density of air is $\boldsymbol{\rho}_\mathbf{a} = 1.1\ \mathrm{kgm^{-3}}$. (All of these numbers are approximate values for room temperature and atmospheric pressure). Find the speed of sound in water and the speed of sound in air. Suppose we had some kryptonite, which in its liquid form, is rather foul smelling. Kryptonite has the density of water but the compressibility of air. What would the speed of sound in kryptonite be?

4. Find the reflection coefficient for a sound wave traveling in air and meeting the water surface (assume one-dimensional motion and perpendicular incidence of the wave on the surface).

5. A transverse sound wave travels along the x-axis in an isotropic solid, medium I. This solid joins a second isotropic solid, medium II, at $\mathbf{x = 0}$. The characteristics of the two solids are such that one quarter of the energy in the incident wave is reflected and three quarters is transmitted into medium II. What can you conclude about the reflection and transmission coefficients **R** and **T**? What can you conclude about the ratio $\mathbf{Z_I/Z_{II}}$ of the impedances of the two materials? (Be careful, there are two solutions.)