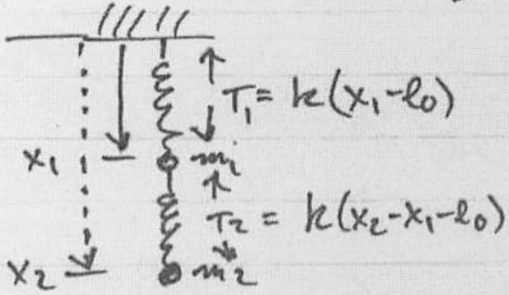
7.0 PH351 HW7

4-5



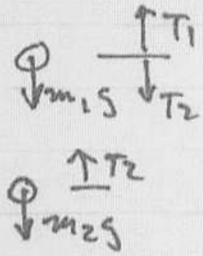
DOING THIS THE <u>HARD WAY</u> :

① ASSUME RESTING LENGTH OF EACH SPRING $= l_0$

② $T_1 \neq T_2$

equation of motion $\quad m_1 \ddot{x}_1 = m_1 g - T_1 + T_2$

$$m_2 \ddot{x}_2 = m_2 g - T_2$$



and $m_1 = m_2$

$$\ddot{x}_1 = g - \frac{k}{m}(x_1 - l_0) + \frac{k}{m}(x_2 - x_1 - l_0)$$

$$\ddot{x}_2 = g - \frac{k}{m}(x_2 - \cancel{l_0}\, x_1 - l_0)$$

the inhomogenous equations are :

$$\ddot{x}_1 + \frac{2k}{m}x_1 - \frac{k}{m}x_2 = g$$

$$\ddot{x}_2 - \frac{k}{m}x_1 + \frac{k}{m}x_2 = g + \frac{kl_0}{m}$$

set $\omega_0^2 = \frac{k}{m}$

we need only solve the homogenous equation as the constant terms set the <u>resting positions</u> of the masses

$$\ddot{x}_1 + 2\omega_0^2 x_1 - \omega_0^2 x_2 = 0$$
$$\ddot{x}_2 - \omega_0^2 x_1 + \omega_0^2 x_2 = 0$$

WHICH YOU ALSO GET BY MEASURING MASS COORDINATES FROM THE <u>EQUILIBRIUM</u> POSITIONS

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4-5 continued.

easiest to use matrix methods.

Assume $x_1 = A_1 \cos\omega t \quad \rightarrow \ddot{x}_1 = -\omega^2 x_1$

$x_2 = A_2 \cos\omega t \quad \rightarrow \ddot{x}_2 = -\omega^2 x_2$

we get

$$(-\omega^2 + 2\omega_0^2)x_1 - \omega_0^2 x_2 = 0$$
$$-\omega_0^2 \; x_1 \; (-\omega^2 + \omega_0^2) x_2 = 0 \quad \Rightarrow A x = 0$$

require $\det A = 0$ $\quad \left[\det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc \right]$

$$\det A = (-\omega^2 + 2\omega_0^2)(-\omega^2 + \omega_0^2) - \omega_0^4 = 0$$

$$\omega^4 - \omega^2\omega_0^2 - 2\omega_0^2\omega^2 + 2\omega_0^4 - \omega_0^4 = 0$$

$$\omega^4 - 3\omega^2\omega_0^2 + \omega_0^4 = 0$$

$$\omega^2 = \frac{3 \pm \sqrt{9-4}}{2}\,\omega_0^2 \quad \Rightarrow \boxed{\omega^2 = \frac{3 \pm \sqrt{5}}{2}\,\omega_0^2}$$

AMPLITUDES : SUBSTITUTE ω^2 BACK INTO EQUATIONS OF MOTION (AND $x_1 = A_1 \cos\omega t$ $\quad x_2 = A_2 \cos\omega t$)

$$\begin{pmatrix} \left(-\frac{3\pm\sqrt{5}}{2} + 2\right)\omega_0^2 & -\omega_0^2 \\ -\omega_0^2 & \left(-\frac{3\pm\sqrt{5}}{2} + 1\right)\omega_0^2 \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

(4-5 cont.)

<u>or</u> $\left(-\frac{3}{2}+\frac{\sqrt{5}}{2}+2\right)A_1 - A_2 = 0$ (faster mode)

$$-A_1 + \left(-\frac{3}{2}+\frac{\sqrt{5}}{2}+1\right)A_2 = 0$$

<u>lower equation gives</u> $-1 + \left(\frac{\sqrt{5}-1}{2}\right)\frac{A_2}{A_1} = 0$

$$\frac{A_2}{A_1} = \frac{1}{\frac{\sqrt{5}-1}{2}} \quad \text{or} \quad \frac{A_1}{A_2} = \frac{\sqrt{5}-1}{2}$$

Slower mode $\left(-\frac{3}{2}-\frac{\sqrt{5}}{2}+2\right)A_1 - A_2 = 0$

$$-A_1 + \left(-\frac{3}{2}+\frac{\sqrt{5}}{2}+1\right)A_2 = 0$$

<u>lower equation:</u> $-1 + \left(-\frac{3}{2}+\frac{\sqrt{5}}{2}\right)\frac{A_2}{A_1} = 0$

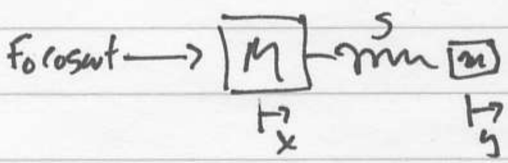
$$\frac{A_2}{A_1} = \frac{-1}{\frac{\sqrt{5}+1}{2}} \quad \text{or} \quad \frac{A_1}{A_2} = -\frac{\sqrt{5}+1}{2}$$

$\boxed{\text{whew !!}}$

GOOD MIDTERM PROBLEM, DON'T YOU THINK?
(JUST KIDDING ...)

$\boxed{\text{REWORDED PROBLEM 4.11}}$ TUNED VIBRATION DAMPERS

- MASS M IS DRIVEN BY FORCE $F_0 \cos \omega t$
- ADD "MASS DAMPER" = SPRING OF STIFFNESS S, AUXILLIARY MASS m



x, y = displacements from equilibrium positions

- ASSUMING $x(t) = A_x \cos \omega t$ AND $y(t) = A_y \cos \omega t$

* SHOW THAT $A_x = 0$ IF $S = \omega^2 m$
* ALSO SHOW THAT $A_y = -\dfrac{F_0}{S}$

SOLUTION: WRITE EQUATIONS OF MOTION

$$M\ddot{x} = -Sx + Sy + F_0 \cos \omega t$$
$$m\ddot{y} = -Sy + Sx$$

or
$$M\ddot{x} + Sx - Sy = F_0 \cos \omega t$$
$$-Sx + m\ddot{y} + Sy = 0$$

Substitute $x = A_x \cos \omega t$ $y = A_y \cos \omega t$, DIVIDE BY $\cos \omega t$

$$-M\omega^2 A_x + SA_x - SA_y = F_0 \qquad (1)$$
$$-SA_x + (S - m\omega^2) A_y = 0 \qquad (2)$$

THE SECOND BECOMES $A_x = \dfrac{(S - m\omega^2) A_y}{S}$ $\boxed{= 0 \text{ IF } S = m\omega^2}$

THE FIRST THEN BECOMES $\boxed{A_y = -F_0/S}$

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(7.1) $c = \sqrt{\frac{T}{\mu}} \Rightarrow$ FOR STRING 1 $c_1 = 2\ m/s = \sqrt{\frac{T_1}{\mu_1}}$

FOR STRING 2 $c_2 = \sqrt{\frac{T_2}{\mu_2}} = \sqrt{\frac{2T_1}{4\mu_1}}$

$\therefore\ c_2 = \frac{c_1}{\sqrt{2}} = 1.41\ m/s$

(7.2) $\frac{T}{\mu}$ has units of $\frac{\text{FORCE}}{\text{mass/LENGTH}} = \frac{Kg\ m/sec^2}{Kg/m}$

$= \frac{m^2}{s^2}$ (VELOCITY)^2

(7.3) Given $\psi(x,t) = 0.1\text{cm} \cos(8s^{-1}t + 4m^{-1}x)$

the frequency $\omega = 8\ s^{-1}$ (radians/sec)

$\nu = \frac{\omega}{2\pi} = \underline{1.27\ Hz}$

wavelength: $k = 4m^{-1} = \frac{\omega}{c} = \frac{2\pi}{\lambda}$

hence $\lambda = \frac{2\pi}{k} = \underline{1.58\ m}$

$c = \frac{\omega}{k} = \frac{8\ s^{-1}}{4m^{-1}} = 2\ m/s$ = $\underline{\text{SPEED OF WAVE}}$

(6)

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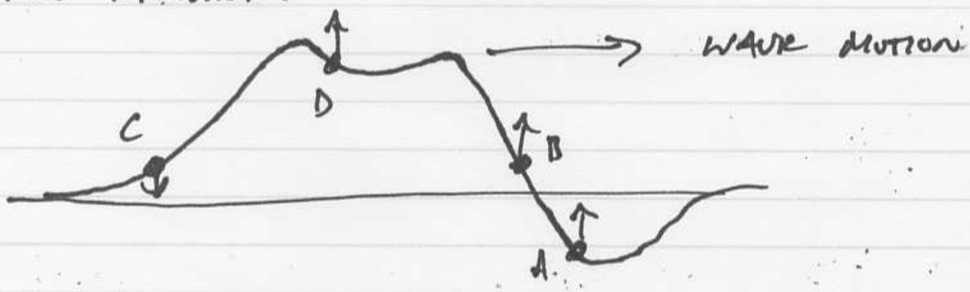
7.4 AS DISCUSSED IN CLASS FOR A WAVE MOVING TO THE RIGHT [of form $f(t - \frac{x}{c})$]:

$$V_y = -c\frac{\partial y}{\partial x} = \frac{\partial y}{\partial t}$$ = transverse velocity of a point

NOTE: FOR A WAVE MOVING LEFT $V_y = +c\frac{\partial y}{\partial x}$

ON THE DIAGRAM



POINT	SLOPE	V
A	(−)	(+)
B	(−)	(+)
C	(+)	(−)
D	(−)	(+)

SLOPE AT (B) > SLOPE AT (A)

SO (B) IS MOVING FASTER THAN (A).