PH 357. HOMEWORK 6 KEY

4.1

$\boxed{\text{TOTAL ENERGY IN } x, y \text{ versus } X, Y}$

(x,y): $KE = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\dot{y}^2$

$$PE = \frac{mg}{2}\frac{x^2}{\ell} + \frac{mg}{2}\frac{y^2}{\ell} + \frac{1}{2}s(x-y)^2$$

$$= \left(\frac{mg}{2\ell} + \frac{s}{2}\right)x^2 - sxy + \left(\frac{mg}{2\ell} + \frac{s}{2}\right)y^2 \quad \to \text{TO 4.2}$$

(X,Y): $E_X = a\dot{X}^2 + bX^2$ (4.3 a)

$E_Y = c\dot{Y}^2 + dY^2$ (4.3 b)

solve for a, b, c, d by comparison ($X = x+y$ $Y = x-y$)

$$E_X = a(\dot{x}^2 + 2\dot{x}\dot{y} + \dot{y}^2) + b(x^2 + 2xy + y^2)$$

$$E_Y = c(\dot{x}^2 - 2\dot{x}\dot{y} + \dot{y}^2) + d(x^2 - 2xy + y^2)$$

so $KE_{XY} = (a+c)\dot{x}^2 + 2(a-c)\dot{x}\dot{y} + (a+c)\dot{y}^2$

$PE_{XY} = (b+d)x^2 + 2(b-d)xy + (b+d)y^2$

PE: $\left\{\begin{array}{l} b - d = -s/2 \\ b + d = \frac{mg}{2\ell} + s/2 \end{array}\right\} \xrightarrow{\text{add subtract}} \left\{\begin{array}{l} b = mg/4\ell \\ d = mg/4\ell + s/2 \end{array}\right.$

KE $\left\{\begin{array}{l} a - c = 0 \\ a + c = \frac{1}{2}m \end{array}\right\} \Rightarrow \left\{\begin{array}{l} a = m/4 \\ c = m/4 \end{array}\right.$

So $E_X = \frac{m}{4}\dot{X}^2 + \frac{mg}{\cancel{4}\ell}X^2$

$E_Y = \frac{m}{4}\dot{Y}^2 + \left(\frac{mg}{\cancel{4}\ell} + \frac{s}{2}\right)Y^2$

NO CROSS TERMS

(4.2) IF INSTEAD WE SUBSTITUTE $X_q = \sqrt{\frac{m}{2}}\,(x+y)$

$Y_q = \sqrt{\frac{m}{2}}\,(x-y)$

$\cancel{PE}$: $E_{X_q} = \frac{am}{2}(\dot{x}^2 + 2\dot{x}\dot{y} + \dot{y}^2) + \frac{bm}{2}(x^2 + 2xy + y^2)$

$E_{Y_q} = \frac{cm}{2}(\dot{x}^2 - 2\dot{x}\dot{y} + \dot{y}^2) + \frac{dm}{2}(x^2 - 2xy + y^2)$

$KE_{X_qY_q} = (a+c)\frac{m}{2}\dot{x}^2 + 2\cdot\frac{m}{2}(a-c)\dot{x}\dot{y} + \frac{m}{2}(a+c)\dot{y}^2$

$PE_{X_qY_q} = (b+d)\frac{m}{2}x^2 + 2\frac{m}{2}(b-d)xy + \frac{m}{2}(b+d)y^2$

PE: $\frac{m}{2}(b-d) = -s$

$\frac{m}{2}(b+d) = \frac{mg}{\cancel{4}\ell} + \frac{s}{2}$

$\Rightarrow b = g/2\ell$

$d = g/2\ell + s/m$

KE $\frac{m}{2}(a-c) = 0$

$\frac{m}{2}(a+c) = \frac{m}{2}$

$\Rightarrow a = 1/2$

$c = 1/2$

$\therefore\ E_{X_q} = \frac{1}{2}\dot{X}_q^2 + \frac{g}{2\ell}X_q^2 = \frac{1}{2}\dot{X}_q^2 + \frac{1}{2}\omega_1^2 X_q^2$

$E_{Y_q} = \frac{1}{2}\dot{Y}_q^2 + \left(\frac{g}{2\ell} + \frac{s}{m}\right)Y_q^2 = \frac{1}{2}\dot{Y}_q^2 + \frac{1}{2}\omega_2^2 Y_q^2$

NO MASSES!

(4.2) THIS WAS ALREADY DONE AS PART OF PROBLEM 4.1 BUT NEEDS TO BE REORDERED

$$E = (KE + PE)_x + (KE + PE)_y + (PE)_{xy}$$

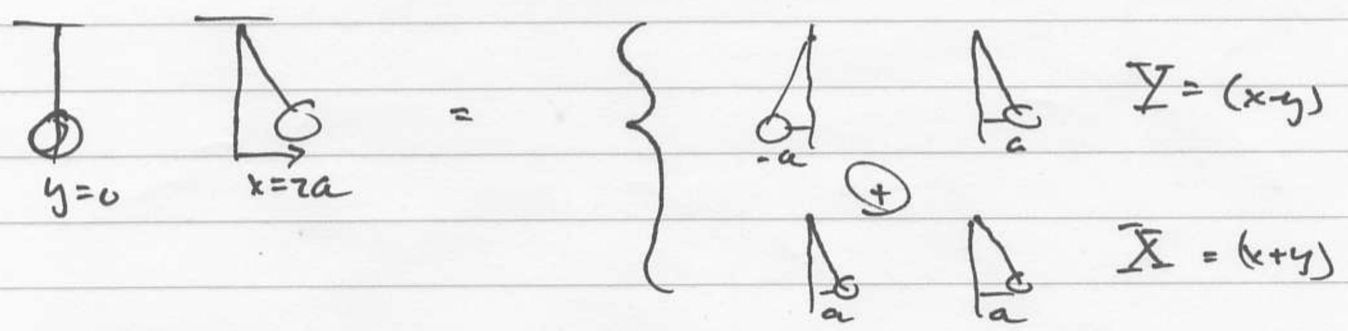
$$= \left(\tfrac{1}{2} m \dot{x}^2 + \left(\tfrac{mg}{2l} + \tfrac{s}{2}\right) x^2\right)$$

$$+ \left(\tfrac{1}{2} m \dot{y}^2 + \left(\tfrac{mg}{2l} + \tfrac{s}{2}\right) y^2\right)$$

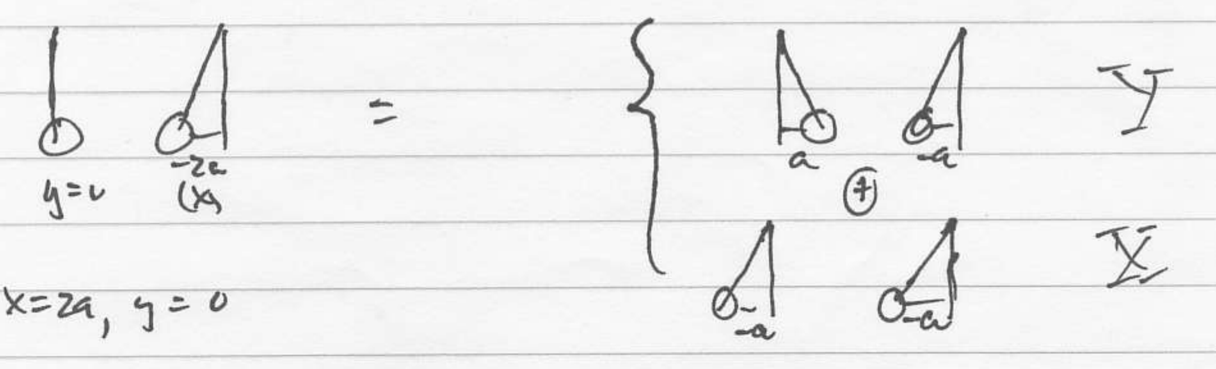
$$+ \left(-sxy\right)$$

THE "COUPLING TERM" $PE_{xy} = -sxy$ $\boxed{\text{and results in exchange of energy}}$

(4.3) DIAGRAMS SUFFICE :



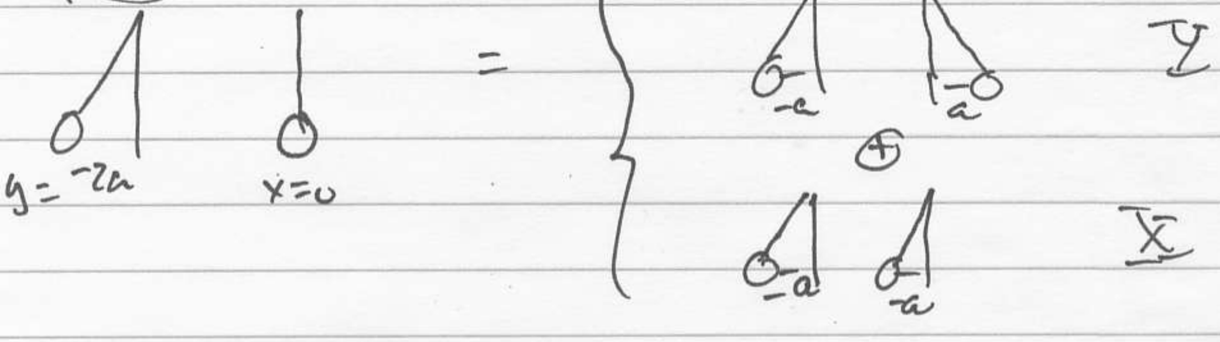
AT LATER TIMES WE GET



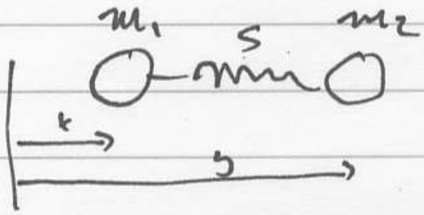
x=2a, y=0

4.3 (CONT)

OR



4.4



$$m_1 \ddot{x} = -sx + sy$$
$$m_2 \ddot{y} = -sy + sx$$

or
$$\ddot{x} + \frac{s}{m_1}x - \frac{s}{m_1}y = 0 \quad (1)$$
$$\ddot{y} - \frac{s}{m_2}x + \frac{s}{m_2}y = 0 \quad (2)$$
SUBTRACT (2) FROM (1)

$$\ddot{x} - \ddot{y} + \left(\frac{s}{m_1} + \frac{s}{m_2}\right)(x-y) = 0$$

define $Y = x - y$ and get $\ddot{Y} + \left(\frac{s}{m_1} + \frac{s}{m_2}\right)Y = 0$

but $s\left(\frac{1}{m_1} + \frac{1}{m_2}\right) = s\left(\frac{m_1 + m_2}{m_1 m_2}\right) = \frac{s}{\mu}$

So $\ddot{Y} + \frac{s}{\mu}Y = 0$ $\boxed{\text{SHM WITH } \omega^2 = \frac{s}{\mu}}$

4.4 (cont)

calculate S from the data

$m_1 = 23 \times 1.67 \times 10^{-27}$ kg $\qquad \omega^2 = (2\pi \cdot 1.14 \times 10^{13}\ Hz)^2$

$m_2 = 35 \times 1.67 \times 10^{-27}$ kg

$$\mu = \frac{(23)(35)}{23+35} \times 1.67 \times 10^{-27}\text{ kg} = 2.3 \times 10^{-26}\text{ kg}$$

$$S = \mu\omega^2 = 2.3 \times 10^{-26}\text{ kg}\,(1.14 \times 10^{13}\ s^{-1})^2 \times (2\pi)^2$$

$$= \underline{119.6\ N/m}$$