Problem 1. From lecture notes, the amplitude response of a driven, damped harmonic oscillator is given by:

\[ A[w_] := \frac{f_0}{\sqrt{\left(w_0^2 - w^2\right)^2 + g^2 w^2}} \]

\[ \phi[w_] := \arctan\left(\frac{w_0^2 - w^2, -g w}{w_0^2 - w^2, g w}\right) \]

Where \( g = \gamma \), \( w = \omega \), \( f_0 = \text{Force constant/mass} \), etc. Put in some constants, i.e. \( m = 0.1 \text{ kg}, s = 1.6 \text{N/m}, b=0.1 \text{ kg/s} \):

\[ w_0 = \sqrt{1.6/0.1}; g = 1.0; f_0 = 2/0.1; \]

\[ \text{Print}["\Omega_0 = \), w0, ", \gamma = \), g] \]
\[ \Omega_0 = 4, \gamma = 1. \]

\[ \text{omega, amplitude, phi = } 0.4, \text{ A[0.4], phi[0.4]} \]
\[ \text{omega, amplitude, phi = } 4, \text{ A[4], phi[4]} \]

\[ \text{omega, amplitude, phi = } 40, \text{ A[40], phi[40]} \]

So, we see that the amplitude is small if \( w_0 \) and \( w \) are different, phases of about 0 or -Pi. Amplitude is large when \( \omega = \omega_0 \), \( \phi = -\pi/2 \)

Problem 2.

The amplitude will be maximum when \( dA/dw = 0 \).

We already entered \( A[w] \), so take its derivative, set it to zero and solve for \( w \).

\[ \text{Clear}[{w0, g, f0}] \]
\[ \text{Solve}\left[D[A[w], w] = 0, w\right] \]

We see from above that if \( \gamma \) is very small, \( A \) is indeed maximum when \( \omega = \omega_0 \), however \( \gamma \) does shift the frequency at which \( A \) is maximum and this is called the "resonant frequency"