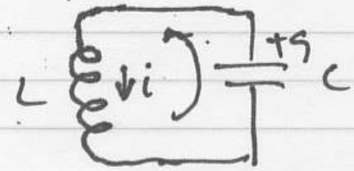
PH 351 HW 3 ①

1



VOLTAGE ACROSS CAPACITOR = $\frac{q}{C}$

VOLTAGE ACROSS INDUCTOR = $L\frac{di}{dt}$

$i = \frac{dq}{dt}$ but since q is ASSUMED to be decreasing for positive i

KIRCHOFF'S LAW: $\Rightarrow i = -\frac{dq}{dt}$ $\quad \frac{di}{dt} = -\frac{d^2q}{dt^2} = -\ddot{q}$

equating VOLTAGE DROPS with VOLTAGE GAINS around loop

$$L\frac{di}{dt} = \frac{q}{C} \Rightarrow L\ddot{q} + \frac{q}{C} = 0 \Rightarrow \boxed{\ddot{q} + \omega_0^2 q = 0}$$

ASSUME SOLUTION OF FORM $q = A\cos(\omega_0 t + \phi)$ with two adj. constants

where $\omega_0 = \frac{1}{\sqrt{LC}}$

checks: $\ddot{q} = -\omega_0^2 A\cos(\omega_0 t + \phi)$

INITIAL CONDITIONS AT $t=0$: $q_0 = CV_1 = A\cos\phi \rightarrow A = \frac{CV_1}{\cos\phi}$

$i = \dot{q} = 0$

$\dot{q} = -\omega_0 A\sin(\omega_0 t + \phi)$

$0 = -\omega_0 A\sin\phi \rightarrow \phi = 0$ AND $\cos\phi = 1$

$\therefore A = CV_1$

$$\boxed{\text{COMPLETE SOLUTION} \quad q(t) = CV_1\cos\frac{t}{\sqrt{LC}}}$$

2

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2 $m = 1.0$ kg $\quad s = 64$ N/m $\quad b$ (frictional coeff) unknown

GIVEN $\ddot{\psi} + \frac{b}{m}\dot{\psi} + \frac{s}{m}\psi = 0$, ASSUME $\psi = e^{pt}$

we found $p = -\frac{b}{2m} \pm \sqrt{\left(\frac{b}{2m}\right)^2 - \omega_0^2}$ where $\omega_0 = \sqrt{\frac{s}{m}}$

UNDAMPED natural frequency $= \sqrt{\frac{64\,N/m}{1\,kg}} = 8\ \text{sec}^{-1}$ $\quad f = \frac{\omega_0}{2\pi} \doteq 1.3$ Hz

This suggests that the circuit would oscillate many times in 10 s if the damping is light,

$\Rightarrow$ take solution to be of form II "light damping"
(NOT ENOUGH INFO TO SOLVE FOR TWO CONSTANTS)

$\psi(t) = A e^{-\frac{bt}{2m}} \cos(\omega_f t + \phi) \qquad \omega_f = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2}$

(a) PROBLEM STATES THAT AMPLITUDE DECREASES FROM A to A/e IN 10 sec

$\therefore \quad \frac{bt}{2m} = 1$ so $b = \frac{2m}{t} = \frac{2 \cdot 1.0\,kg}{10\,s}$ $\boxed{= 0.2\ kg/s}$

(b) $\gamma = \frac{b}{m} = 0.2\ s^{-1}$ $\quad \boxed{Q = \frac{\omega_0}{\gamma} = \frac{8}{0.2} = 40}$

(c) CRITICAL DAMPING IS DEFINED AS $\omega_f = 0 = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2}$

$\omega_0^2 = \left(\frac{b}{2m}\right)^2$ or $\gamma^2 = 4\omega_0^2$ so $\boxed{Q = 1/2}$

$\boxed{b = 2m\omega_0 = 16\ kg\,s^{-1}}$

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- **Damped SHM equation** CHECK OF RESULT PROBLEM 2 part (a)

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result1 = NDSolve[{y''[t] + 0.2 y'[t] + 8 y[t] == 0, y[0] == 1, y'[0] == 0}, y, {t, 0, 10}]
{{y → InterpolatingFunction[{{0., 10.}}, <>]}}
plot1 = Plot[y[x] /. result1, {x, 0, 10}];
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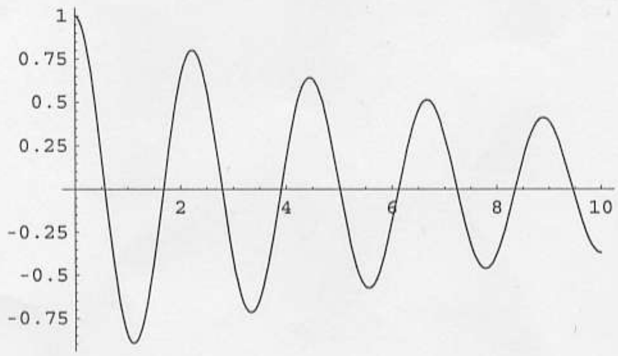

3. $\psi = A e^{-\gamma t/2} \cos(\omega_f t + \phi)$

AMPLITUDE IS GIVEN BY $A e^{-\gamma t/2}$

AT TIME t this decreases by $\frac{1}{2}$

$\frac{1}{2} = e^{-\gamma t/2} \Rightarrow \ln(0.5) = -\gamma t/2$

$t = -2\frac{\ln(0.5)}{\gamma} = \frac{1.39}{\gamma}$

4. $\psi(t)$ AS ABOVE

THE COS TERM GIVES SUCCESSIVE MAXIMA FOR $T = \frac{2\pi}{\omega_f}$

THIS IS WHAT THE PROBLEM ASKS US TO SHOW SO

WHY DOESN'T $e^{-\gamma t/2}$ TERM AFFECT THE TIME BETWEEN MAXIMA? (It does affect the position of the maxima, see FRENCH)

AT $\frac{\text{MAX}}{\text{MIN}}$ OF $\psi(t)$, $\dot{\psi}(t) = 0 = -\frac{\gamma}{2} t A e^{-\gamma t/2} \cos(\omega_f t + \phi) - \omega_f A e^{-\gamma t/2} \sin(\omega_f t + \phi)$

THIS LEADS TO $\frac{\sin(\omega_f t + \phi)}{\cos(\omega_f t + \phi)} = \tan(\omega_f t + \phi) = -\gamma / 2\omega_f$

THIS IS PERIODIC IN t WITH PERIOD $T = \frac{2\pi}{\omega_f}$ (for maxima)

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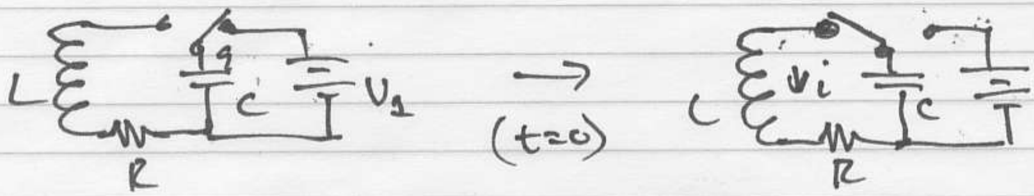
④

⑤ RLC CIRCUIT



$$L\frac{di}{dt} + Ri + \frac{q}{C} = 0 \qquad q_0 = CV_1$$

$$i = \dot{q} \quad \frac{di}{dt} = \ddot{q} \quad \Rightarrow \quad \ddot{q} + \frac{R}{L}\dot{q} + \frac{q}{C} = 0 \quad \begin{cases} \omega_0 = \frac{1}{\sqrt{LC}} \\ \omega_f = \sqrt{\omega_0^2 - \left(\frac{\gamma}{2}\right)^2} \end{cases}$$

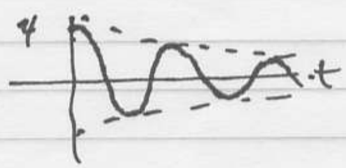
ASSUMING $q = A\cos(\omega_f t + \phi) \cdot e^{-\gamma t/2}$

$$q(0) = CV_1 = A\cos\phi \quad \Rightarrow \quad A = \frac{q(0)}{\cos\phi} = \frac{CV_1}{\cos\phi}$$

$$\dot{q}(t) = -\frac{\gamma}{2}A e^{-\gamma t/2}\cos(\omega_f t + \phi) - \omega_f A e^{-\gamma t/2}\sin(\omega_f t + \phi)$$

$i=0$ WHEN SWITCH CLOSED At $t=0$ $\longrightarrow$ $\dot{q}(0) = 0 = -\frac{\gamma}{2}A\cos\phi - \omega_f \sin\phi$

$$\Rightarrow \tan\phi = \frac{-\gamma}{2\omega_f}$$



(a) SO $q(t) = \frac{CV_1}{\cos\phi} e^{-\gamma t/2}\cos(\omega_f t + \phi)$ where $\phi = \tan^{-1}\left(\frac{-\gamma}{2\omega_f}\right)$

✱ SURPRISINGLY A IS NOT EQUAL TO $q(0)$. THIS IS BECAUSE THE RESISTOR INTRODUCES A TIME LAG INTO THE SOLUTION. CHECK: $q(0) = \frac{CV_1}{\cos\phi} \cdot \cos\phi = CV_1$

(b) CRITICAL DAMPING FOR $\gamma^2 = 4\omega_0^2$ OR $\left(\frac{R}{L}\right)^2 = 4\left(\frac{1}{LC}\right)$

$\boxed{\text{OSCILLATES}}$ FOR $\boxed{R < 2\sqrt{L/C}}$