VOLTAGE ACROSS CAPACITOR: \( \frac{q}{C} \)

VOLTAGE ACROSS INDUCTION: \( L \frac{di}{dt} \)

\( i \frac{dq}{dt} \) but since \( q \) is assumed to be decreasing for positive \( i \)

KIRCHHOFF'S LAW: \[ \Rightarrow \frac{dq}{dt} \frac{di}{dt} = -\frac{dq}{dt} = -\frac{dq}{dt} \]

Equating voltage drops with voltage gains around loop:

\[ \frac{Ldi}{dt} = \frac{q}{C} \Rightarrow L\dot{q} + \frac{q}{C} = 0 \Rightarrow \dot{q} + \frac{1}{LC}q = 0 \]

Assume solution of form \( q = A \cos (\omega_0 t + \phi) \) with two adjustable constants

where \( \omega_0 = \frac{1}{\sqrt{LC}} \)

Checks: \( \dot{q} = -\omega_0 A \cos (\omega_0 t + \phi) \)

Initial conditions:

\[ q = CV_1 = A \cos \phi \Rightarrow A = \frac{CV_1}{\cos \phi} \]

\[ i = \dot{q} = 0 \]

\[ \ddot{q} = -\omega_0^2 A \sin (\omega_0 t + \phi) \]

\[ 0 = -\omega_0^2 A \sin \phi \rightarrow \phi = 0 \text{ and } \cos \phi = 1 \]

\[ \therefore A = CV_1 \]

Complete solution:

\[ q(t) = CV_1 \cos \frac{t}{\sqrt{LC}} \]
2. \( m = 1.0 \text{ kg} \) \( S = 64 \text{ N/m} \) \( b \) (frictional coefficient) is unknown.

**Given** \( \ddot{y} + \frac{k}{m} \dot{y} + \frac{S}{m} y = 0 \) \( \text{Assume} \ y = e^{pt} \)

We found \( p = \frac{-b}{2m} \pm \sqrt{(\frac{b}{2m})^2 - \omega_0^2} \) where \( \omega_0 = \sqrt{\frac{S}{m}} \)

**Undamped natural frequency** \( = \sqrt{\frac{64 \text{ N/m}}{1 \text{ kg}}} = 8 \text{ sec}^{-1} \)

\( \omega_f = \frac{\omega_0}{2\pi} \approx 1.3 \text{ Hz} \)

This suggests that the circuit would oscillate many times in 10 s if the damping is light.

\[ y(t) = A e^{\frac{-bt}{2m}} \cos(\omega_f t + \phi) \]

\( \omega_f = \sqrt{\omega_0^2 - (\frac{b}{2m})^2} \)

(a) **Problem states that amplitude decreases from \( A \) to \( A/e \)**

in 10 sec:

\[ \frac{6e}{2m} = 1 \quad \text{so} \quad 6 = \frac{2m}{t} = \frac{2 \cdot 1.016}{10} \]

\[ \therefore t = 0.2 \text{ s} \]

(b) \( \delta = \frac{b}{\sqrt{2m}} = 0.2 \text{ s}^{-1} \)

\[ Q = \frac{\omega_0}{\delta} = \frac{8}{0.2} = 40 \]

(c) **Critical damping is defined as** \( \omega_f = 0 = \sqrt{\omega_0^2 - (\frac{b}{2m})^2} \)

\[ \omega_0^2 = (\frac{b}{2m})^2 \] or \( \delta^2 = 4\omega_0^2 \) \( \therefore \]

\[ Q = \frac{1}{2} \]

\[ b = 2\pi\omega_0 = 16 \text{ kg s}^{-1} \]
Damped SHM equation

Check of Result

Problem 12 part a

\[ \text{result1 = NDSolve}[\{y''[t] + 0.2 y'[t] + 8 y[t] = 0, y[0] = 1, y'[0] = 0\}, y, \{t, 0, 10\}] \]
\[ \{(y \rightarrow \text{InterpolatingFunction}[\{(0., 10.\}, \ldots])\} \] 

\[ \text{plot1 = Plot}[y[x] /. \text{result1}, \{x, 0, 10\}]; \]

3. \[ y = Ae^{-\frac{t}{2}} \cos(\omega_f t + \phi) \]

Amplitude is given by \( A \), \( e^{-\frac{t}{2}} \)

At time \( t \) this decays by \( \frac{1}{2} \)

\( I = e^{-\frac{t}{2}} \Rightarrow I(0.1) = -0.5t \)

\[ t = -2 \ln(0.5) = 1.39 \]

4. \( \psi(t) \) at above

The \( \cos \) term gives successive maxima for \( T = \frac{2\pi}{\omega_f} \)

This is what the problem asks us to show so

Why doesn't \( e^{-\frac{t}{2}} \) term affect the time between maxima? (It does affect the position of the maxima, see figure)

At max of \( \psi(t) \), \( \psi(t) = 0 = -\frac{\pi}{2} + \frac{\pi}{2} \cos(\omega_f t + \phi) - \frac{\pi}{2} e^{-\frac{t}{2}} \sin(\omega_f t + \phi) \)

This leads to \( \frac{\sin(\omega_f t + \phi)}{\cos(\omega_f t + \phi)} = \tan(\omega_f t + \phi) = -\frac{\pi}{2\omega_f} \)

This is periodic in \( t \) with period \( T = \frac{2\pi}{\omega_f} \) (for maxima)
HW 3

RLC circuit

\[ L \frac{di}{dt} + Ri + \frac{q}{C} = 0 \quad \phi_0 = CV_1 \]

\[ i = \Phi \quad \frac{di}{dt} = \dot{\Phi} \quad \Rightarrow \quad \ddot{\Phi} + \frac{R}{L} \dot{\Phi} + \frac{1}{C} \Phi = 0 \]

Assuming \( \Phi = A \cos (\omega t + \phi) e^{-\alpha t} \)

\[ \Phi(0) = CV_1 \quad \Rightarrow \quad A = \frac{CV_1}{\cos \phi} \]

\[ \Phi'(0) = -\frac{A}{2} e^{-\alpha t} \cos (\omega t + \phi) - \alpha A e^{-\alpha t} \sin (\omega t + \phi) \]

\[ \Phi(0) = 0 = -\frac{A}{2} \cos \phi - \omega t \sin \phi \]

\[ \Rightarrow \tan \phi = -\frac{\alpha}{2 \omega} \]

(a) So \( \Psi(t) = \frac{CV_1}{\cos \phi} e^{-\alpha t} \cos (\omega t + \phi) \) where \( \phi = \tan^{-1}(\frac{-\alpha}{2 \omega}) \)

\[ \text{Surprisingly, } A \text{ is not equal to } \Phi(0). \text{ This is because the resistor introduces a time lag into the solution. Check: } \Phi(0) = \frac{CV_1}{\cos \phi} \cos \phi = CV_1 \]

(b) Critical damping for \( \gamma^2 = 4 \omega^2 \) or \( \left( \frac{\gamma}{\omega} \right)^2 = 4 \left( \frac{1}{\gamma \omega} \right) \)

\[ R \ll 2 \sqrt{\frac{L}{C}} \]