PH 351 HW 2

1 (a) $\frac{1}{i} = \frac{\{1,0\}}{\{0,1\}} = \left\{ \frac{1\cdot 0 + 0\cdot 1}{0^2+1^2}, \frac{0\cdot 0 - 1\cdot 1}{0^2+1^2} \right\} = \{0,-1\} = -i$

$\boxed{\text{or}}$ $\frac{1}{i} = -\frac{(i\cdot i)}{i} = \underline{\underline{-i}}$ $\boxed{\text{or}}$ $\frac{1}{i} = \frac{1}{e^{i\pi/2}} = e^{-i\pi/2} = \underline{\underline{-i}}$

(b) $e^{i\pi} = \cos\pi + i\sin\pi = -1 + 0i = \underline{\underline{-1}}$

(c) $e^{i\pi/2} = \cos\pi/2 + i\sin\pi/2 = 0 + 1i = \underline{\underline{i}}$

(d) $i^i = e^{i\pi/2\cdot i} = \boxed{e^{-\pi/2} \sim 0.208}$

$\boxed{\text{or}}$ $z = i^i \rightarrow \ln z = i \ln i$

$\ln z = i \ln e^{i\pi/2} = i\cdot i\pi/2 = \underline{\underline{-\pi/2}}$

$\Rightarrow z = e^{-\pi/2}$

2 $z^2 + 2z + 2 = 0$

$z = \frac{-b \pm \sqrt{b^2 - 4\cdot a\cdot c}}{2a} = \frac{-2 \pm \sqrt{2^2 - 4\cdot 1\cdot 2}}{2}$

$= -1 \pm \sqrt{-4} = \boxed{-1 \pm 2i}$

PH 351 HW 2 ②

③ $m = 2.0$ kg $\quad S = 8$ N/m $\quad A = 0.1$ m $\quad \phi = \pi/2$

$$\Rightarrow \omega_0 = 2\ s^{-1}$$

BEGIN WITH $\psi(t) = 0.1 \cos(2t + \pi/2)$

(a) use $\cos(u+v) = \cos u \cos v - \sin u \sin v$

$$\psi(t) = 0.1\left(\cos 2t \cos \pi/2 - \sin 2t \sin \pi/2\right)$$

$$= -0.1 \sin 2t$$

$$\boxed{B_p = 0, \quad B_q = -0.1}$$

(b) use $\cos\phi \equiv \frac{1}{2}\left(e^{i\phi} + e^{-i\phi}\right)$

write $\psi(t) = \frac{A}{2}\left(e^{i(\omega_0 t + \phi)} + e^{-i(\omega_0 t + \phi)}\right)$

$$= \frac{A}{2} e^{i\phi} e^{i\omega_0 t} + \frac{A}{2} e^{-i\phi} e^{-i\omega_0 t}$$

$$= C e^{i\omega_0 t} + C^* e^{-i\omega_0 t}$$

where $C = \frac{0.1}{2} \cdot e^{i\pi/2} = 0.05 i$

hence $\psi(t) = 0.05 i e^{i\omega_0 t} - 0.05 i e^{-i\omega_0 t}$ $\quad (\omega_0 = 2)$

(c) we have $A\cos(\omega_0 t + \phi) = Re\left(D e^{i\omega_0 t}\right)$ so D must be complex $= D e^{i\phi_0}$

$$= Re\left[|D| e^{i\phi_D} e^{i\omega_0 t}\right]$$

conclude $\psi(t) = Re\left[0.1 i e^{i2t}\right]$ $\quad D = 0.1 i = 0.1 e^{i\pi/2}$

(4) HW 2

ON A PHONOGRAPH RECORD THE GROOVES "WIGGLE" AND MOVE THE STYLUS IN A CLOSE APPROXIMATION TO THE AMPLITUDE OF THE SOUND WAVE

$\psi = A \cos \omega t$ here, $A = 0.01\ mm = 10^{-5}\ m$

$\omega = 2\pi f = 2\pi \cdot 2000\ Hz$

$= 4000\pi\ s^{-1}$

acceleratin $= \ddot{\psi} = -\omega_0^2 A \cos \omega t$, Max value $= \omega_0^2 A$

$= (4000\pi\ s^{-1})^2 \cdot 10^{-5}\ m = 1{,}577\ m/s^2$

interms of "g" acc $= \dfrac{1{,}577}{9.8 (m/sec^2)} \cong 160\ g$

(5)

$\omega_0 = \sqrt{\dfrac{s}{m}}$ which is independent of gravity

(a) FIND SPRING CONSTANT s : $F = -ks = mg$

HAVE $1 kg \cdot 9.8 m/s^2 = 100\ mm = 0.1\ m$

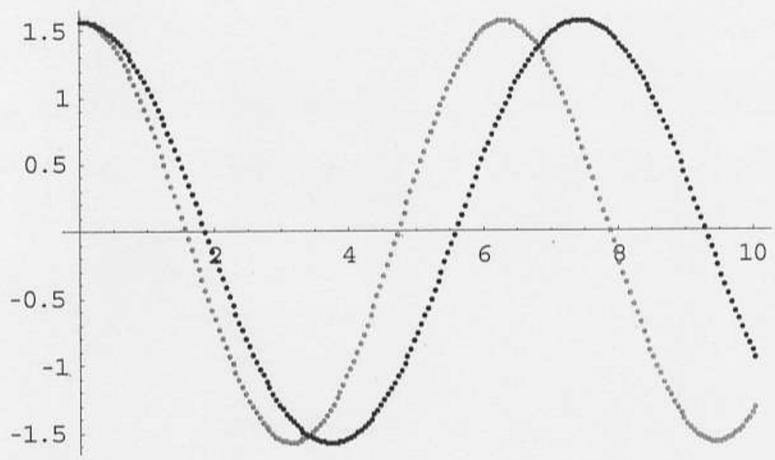
$\Rightarrow s = 98\ N/m$

(b) $\omega_0^2 = \dfrac{s}{m} \Rightarrow m = \dfrac{s}{\omega_0^2}$ HAVE $T = 1\ s$ So $\omega_0 = \dfrac{2\pi}{T} = 2\pi\ s^{-1}$

$m = \dfrac{98\ N/m}{(2\pi)^2 sec^{-2}} = 2.48\ Kg$

SINCE weight = mg, $\dfrac{\text{"g" on the moon}}{\text{"g" on the earth}} = \dfrac{0.4\ kg}{2.48\ kg} = 0.16 \sim \dfrac{1}{6}$ THAT OF EARTH

PH351 Hw2 – problem 6



Using the program RungeKutta2.nb (posted), Setting m=g=l=1, starting x0 = pi/2, v0=0 the results of the numerical simulation are as shown above.

Cyan: F(x) = -x, Pink: F(x) = -sin(x)

Period for linear force ~ 6.2 s (should be 2pi)
Period for nonlinear force ~ 7.4 s

Why ~~is~~ does the nonlinear force have the longer period?