PH351 HW 10 (OPTIONAL)

① FOR BARELY AUDIBLE SOUND @ 500 Hz, $I_0 \sim 10^{-12}\ W/m^2$

$$I = \frac{1}{2} Z \omega^2 A^2$$

FOR AIR $Z = \rho_0 c = 1.2\ kg/m^3 \times 340\ m/s$ @ 20°C

$$\approx 408\ \frac{kg}{m^2 s}$$

$\therefore A$ = AMPLITUDE OF DISPLACEMENT $= \sqrt{\frac{2 I_0}{\omega^2 Z}}$

$$A = \sqrt{\frac{2 \cdot \times 10^{-12}\ W/m^2}{(2\pi \times 500\ s^{-1})^2 \cdot 408\ \frac{kg}{m^2 s}}} = 2 \times 10^{-11}\ m$$

AN ATOM IS $\sim 10^{-10}$ m IN DIAMETER

② $10^{12}\ I_0 \sim 1\ W/m^2 \times 3m \times 3m \sim 9\ W$

THIS IS ACOUSTIC POWER, NOT AMPLIFIER POWER. SPEAKERS ARE VERY INEFFICIENT AT CONVERTING ELECTRICAL TO SOUND ENERGY.

③ FROM PAIN CH 8 (p 220 in 6^{th} ED)

$R \sim \frac{n_1 - n_2}{n_1 + n_2}$ where $n = \frac{c}{v} = \frac{\text{SPEED OF LIGHT IN VACUO}}{\text{" " " IN MATERIAL}}$

$R_{AIR \to WATER} = \frac{n_{air} - n_{water}}{n_{air} + n_{water}} = \frac{1 - 1.33}{1 + 1.33} \approx -0.14$ (SIGN IS IMMATERIAL)

$\frac{I_R}{I_i} = R^2 = 0.02$ (2% OF ENERGY IS REFLECTED).

WORKS IN EITHER DIRECTION SO $R_{WATER \to AIR} = 0.14$

$I_R/I_i = 0.02$

HW 10 PROBLEM 4

$\boxed{\text{6.16 PAIN}}$ POORLY PHRASED BUT HERE IS ONE INTERPRETATION

- THE HYDROSTATIC PRESSURE OF WATER IS THE PRESSURE EXERTED BY THE WEIGHT OF THE WATER COLUMN ABOVE THE POINT OF INTEREST.

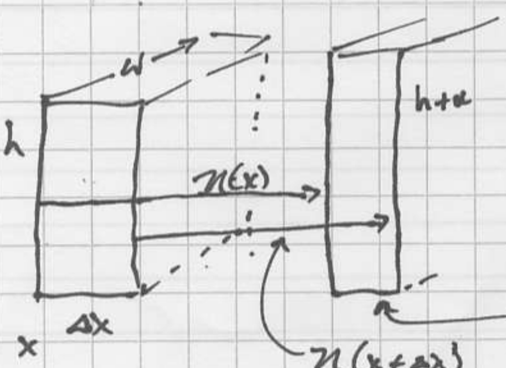
CONSIDER SLAB OF WATER OF HEIGHT h, width Δx, length w

$$\text{WEIGHT} = mg = \rho V g = \rho h w \Delta x g$$

$$\text{PRESSURE} = \frac{F}{A} = \rho \frac{h w \Delta x g}{w \Delta x} = \rho g h = P$$

(P POINTS IN ALL DIRECTIONS)

NOW: SLAB MOVES a distance η, $\left\{ \begin{array}{l} \text{height increases to } h+\alpha \\ \text{width decreases} \end{array} \right.$



(SLAB MOVES TO RIGHT)

$$\text{new width} = \Delta x + \left(\eta(x+\Delta x) - \eta(x)\right) \cdot \frac{\Delta x}{\Delta x}$$

$$= \Delta x + \frac{\partial \eta}{\partial x} \Delta x$$

$$\text{NEW VOLUME} = (h+\alpha) w \left(\Delta x + \frac{\partial \eta}{\partial x} \Delta x\right)$$

$$= (h+\alpha) w \Delta x \left(1 + \frac{\partial \eta}{\partial x}\right)$$

$= \underline{\text{OLD VOLUME}}\ (h w \Delta x)$ BECAUSE WATER ~ INCOMPRESSIBLE

$$h \cancel{w} \cancel{\Delta x} = (h+\alpha) \cancel{w} \cancel{\Delta x} \left(1 + \frac{\partial \eta}{\partial x}\right)$$

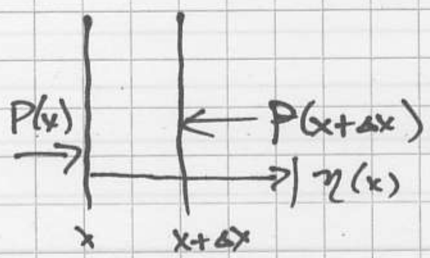
$$h = (h+\alpha)\left(1 + \frac{\partial \eta}{\partial x}\right)$$

$\dfrac{1}{1+\frac{\alpha}{h}} = 1 + \dfrac{\partial \eta}{\partial x}$ BUT IF α/h SMALL $\left(1+\frac{\alpha}{h}\right)^{-1} \sim 1 - \alpha/h$

$$\Rightarrow \boxed{\alpha = -h \, \partial \eta / \partial x} \quad (1)$$

(4) CONT.

SLAB IS SUBJECTED TO PRESSURE FROM RIGHT AND LEFT



$$\boxed{ma = F}$$

$$\rho h w \Delta x \frac{\partial^2 \eta}{\partial t^2} = -hw\left(p(x+\Delta x) - p(x)\right)\frac{\Delta x}{\Delta x}$$

$$= -hw\frac{\partial P}{\partial x}\Delta x$$

(THIS ONLY HOLDS AT A GIVEN {DEPTH / HEIGHT} BUT THE EQUATION HAS THE SAME FORM AT ALL {HEIGHTS / DEPTHS})

→ should integrate from 0 to h TO AVERAGE OVER COLUMN

BUT CARRY ON!

$$\Rightarrow \rho \frac{\partial^2 \eta}{\partial t^2} = -\frac{\partial P}{\partial x}$$ TRUE FOR ANY WAVE IN MATTER

P DEPENDS ON {DEPTH / HEIGHT} $= P_0 + \delta P$

$$= P_0 + \rho g\,(h + \alpha)$$

$$\rho \frac{\partial^2 \eta}{\partial t^2} = \frac{-\partial}{\partial x}\left(P_0 + \rho g(h+\alpha)\right)$$

$$= \rho g\left(-\frac{\partial \alpha}{\partial x}\right)$$

$$= \rho g\left(-\frac{\partial}{\partial x}\left(-h\frac{\partial \eta}{\partial x}\right)\right)$$ from (1)

SO $$\frac{\partial^2 \eta}{\partial t^2} = gh\frac{\partial^2 \eta}{\partial x^2}$$

WAVE EQUATION

VELOCITY $c^2 = gh$

FINAL NOTE: P_0 = ATMOSPHERIC PRESSURE ON TOP OF SLAB. ENORMOUS!!