Name Key

Physics 351
Foundations of Physics II
Final Exam, 7 December 2005

This is a closed book exam, except that one page of notes is permitted. Please put your name on each page (in case the pages become separated). If you have any questions about the problems, please don't hesitate to ask! **Please show your work.**

1. There are a few physical situations in which an object is subjected to a "reverse spring force". Mathematically, consider a particle of mass **m** moving along the **x** axis, subjected to a force **F = +sx** (where **s** is the "stiffness"). Friction is ignored, so the motion of the particle obeys the differential equation

$$m\ddot{x} - sx = 0$$

(a) 10] Show how to solve this equation for **x(t)** and give the general solution, which should have two adjustable constants, A and B, in addition to the physical constants **s** and **m**. Does the particle oscillate?

(b) [10] Find expressions which relate A and B to the constants **s** and **m** as well as the initial position ($\mathbf{x_0}$) and velocity ($\mathbf{v_0}$) at t=0.

(c) [15] For certain values of A and B, the particle comes to rest after a long time, that is ($t \to \infty$). For what values of A and B is this true? What constraints does this condition place on the initial values of $\mathbf{x_0}$ and $\mathbf{v_0}$?

(a) ASSUME $x(t) = e^{pt}$, $\ddot{x}(t) = p^2 e^{pt}$ D.E. BECOMES $mp^2e^{pt} - se^{pt} = 0$

OR $mp^2 - s = 0$

SO $p = \pm\sqrt{s/m} \equiv \pm\omega$

GENERAL SOLUTION: $x(t) = Ae^{\omega t} + Be^{-\omega t}$ <u>NOT OSCILLATORY</u>

(b) $x(0) = A + B = x_0$

$\dot{x}(0) = +\omega A - \omega B = v_0$

$A + B = x_0$ ①

$A - B = v_0/\omega$ ②

ADD: ①+② $2A = x_0 + v_0/\omega$, $\underline{A = \frac{1}{2}(x_0 + v_0/\omega)}$

SUBTRACT ①−② $2B = x_0 - v_0/\omega$, $\underline{B = \frac{1}{2}(x_0 - v_0/\omega)}$

(c) DOES NOT BLOW UP IF $\underline{A = 0}$ B CAN BE ANYTHING. $\boxed{y = Be^{-\omega t} \to 0 \text{ as } t \to \infty}$

THIS REQUIRES $0 = x_0 + v_0/\omega$ OR $\boxed{v_0 = -x_0\omega}$

<u>SEVERE RESTRICTION</u> ON ALLOWABLE VALUES!

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2. Suppose that a sound wave emitted by a depth finder on a fishing boat comes to a layer of toxic sludge (*X*) secretly dumped into the lake by a certain large chemical corporation. From the unusual intensity of the echo, you suspect that *X* has the same compressibility as water $\kappa_w = 4.9 \times 10^{-10}\ \mathrm{m^2\,N^{-1}}$ (bulk modulus $B = 1/\kappa_w$) but four times the density, $\rho(X) = 4.0 \times 10^3\ \mathrm{kg\,m^{-3}}$. The propagation direction of the sound wave is perpendicular to the interface between the water and the sludge *X*.

(a) [10] Find the speed of sound in the water and in the sludge.
(b) [15] What fraction of the incident sound intensity is reflected?

$$c_w = \sqrt{\frac{1}{\kappa_w \rho_w}} = \sqrt{\frac{1}{4.9\times10^{-10}\,m^2N^{-1}\cdot 1.0\times10^3\,kg/m^3}} = 1428\ m/s$$

$$c_x = \sqrt{\frac{1}{\kappa_w \cdot 4\rho_w}} = \frac{c_w}{2} = 714\ m/s$$

$$Z_w = \rho_w c_w = 1.0\times10^3\,\frac{kg}{m^3}\cdot 1428\,m/s = 1.43\times10^6\,\frac{kg}{m^2 s}$$

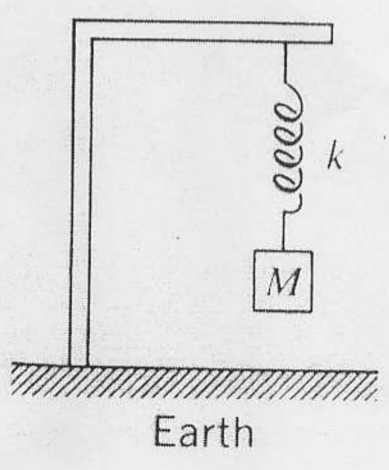
$$Z_x = 4\rho_w\cdot\frac{c_w}{2} = 2.86\times10^6\,kg/m^2s = 2Z_w$$

$$\frac{I_R}{I_i} = \left(\frac{Z_1-Z_2}{Z_1+Z_2}\right)^2 = \left(\frac{1-2}{1+2}\right)^2 = \left(\frac{-1}{3}\right)^2 = 0.11$$

OR 10% INTENSITY REFLECTED

3. A typical long-period seismometer, used to detect earthquakes, is basically a vertically suspended spring and mass with a period of oscillation of 30.0 seconds and a **Q** of 2.0 Recall that $\mathbf{Q} = \omega_0/\gamma$. It is thus a driven damped oscillator, where the driving force is supplied by the vertical motion of the earth's surface.

$$\boxed{\gamma = \frac{\omega_0}{2}}$$



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(a) [10] Accurately calculate the angular frequency ω_0 of oscillation of the seismometer.
(b) [5] Sketch a graph of the amplitude **A** of the vertical displacement **y** of the seismometer as a function of the frequency of oscillation ω of the earth's surface.
(c) [5] As a result of an earthquake, the earth's surface oscillates vertically with a period of 20 minutes and amplitude such that the maximum acceleration is 10^{-9} m/sec^2. What is the amplitude of oscillation of the earth's surface?
(d) [15] How small a value of A must be observable if this event is to be detected by the seismometer?

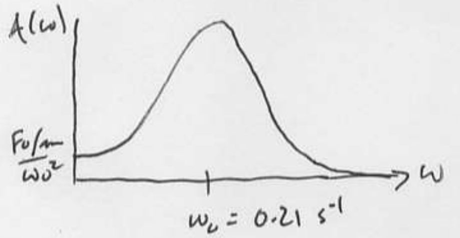
(a) $\omega_s = \sqrt{\omega_0^2 - \left(\frac{\gamma}{2}\right)^2}$ BUT $\gamma = \frac{\omega_0}{2}$ SO $\omega_s = \sqrt{\omega_0^2 - \left(\frac{\omega_0}{4}\right)^2}$

$\omega_0 = \sqrt{k/M}$ $\qquad = \omega_0\sqrt{1-\frac{1}{16}}$

PERIOD $T = \frac{2\pi}{\omega_s}$ SO $\omega_s = \frac{2\pi}{30} = 0.2094\ s^{-1}$ $\quad \omega_s < \omega_0$!

$\omega_0 = \frac{\omega_s}{\sqrt{15/16}} = 0.2163\ s^{-1}$

(b)



$$A(\omega) = \frac{F_0/m}{\sqrt{(\omega_0^2-\omega^2)^2+\gamma^2\omega^2}}$$

= AMPLITUDE OF SEISMOMETER OSCILLATIONS

(c) $\ddot{y} = -\omega_{EARTH}^2 A_{EARTH} = 10^{-9}\ m/s^2$

SO $|A_{EARTH}| = \frac{10^{-9}\ m/s^2}{\omega_{EARTH}^2}$

$\omega_{EARTH} = \frac{2\pi}{T} = \frac{2\pi}{(20\ min)(60\ s/min)}$

$= 0.0052\ s^{-1}$

$A_{EARTH} = \frac{10^{-9}\ m/s^2}{(0.0052\ s^{-1})^2} = 3.6 \times 10^{-5}\ m$

(d) $\boxed{A(\omega) \sim \frac{F_0/m}{\omega_0^2} \quad \text{SINCE} \quad \omega_0 >> \omega}$

$A_{SEISMOMETER} \sim \frac{10^{-9}\ m/s^2}{(0.216)^2 s^{-2}} = 2.14 \times 10^{-8}\ m$ VERY SMALL OSCILLATIONS !

CHECK : $\gamma^2\omega^2 = \frac{\omega_0^2}{4}\cdot\omega^2 = \left(\frac{0.216}{2}\right)^2\cdot(0.0052)^2 = 2.7\times10^{-5}\ s^{-4}$

$(\omega_0^2-\omega^2)^2 = ((.216)^2-(0.0052)^2)^2 = 2.2\times10^{-3}$

$\omega_0^4 = 2.2\times10^{-3}$

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4. A long uniform string of mass density 0.1 kg/m is stretched with a tension of 50N. One end of the string (at **x** = **0**) is wiggled transversely (and sinusoidally) with an amplitude of 0.02 m and a period of 0.1 sec, so that traveling waves are set up in the +**x** direction.

(a) [5] What is the velocity of the waves?

(b) [5] What is their wavelength?

(c) [15] If at the driven end (**x** = **0**), the displacement at **t** = **0** is 0.01 m, find the complete equation describing the traveling waves.

0.02

(a) $c = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{50\,N}{0.1\ kg/m}} = 22.4\ m/s$

(b) $c = \lambda f$ so $\lambda = \frac{c}{f} = \frac{22.4\ m/s}{1/T} = 22.4\ m/s \times 0.1\ s$

$= 2.24\ m$

(c) ASSUMING $y(t) = A \cos(\omega t - kx)$

$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.1\ s} = 62.8\ s^{-1}$

$k = \frac{2\pi}{\lambda} = \frac{\omega}{c} = 2.8\ m^{-1}$

[check $\lambda = \frac{2\pi}{k} = \frac{2\pi}{2.8\ m^{-1}} = 2.24\ m$]

$A = 0.02$ SINCE $t=0$ and $x=0$

$\therefore\ y(t) = 0.02 \cos\left(62.8\ s^{-1}\,t - 2.8\ m^{-1}\,x\right)$ SATISFIES BOUNDARY COND.

{NOTE TYPO IN PROBLEM → POSSIBLE CONFUSION WITH $A = 0.01\ m$}

Have a nice break! OK!

CLASSIC CALVIN AND HOBBES